LONGEVITY RISK AND LONG-TERM CARE: ANNUITIES, LONG TERM CARE INSURANCE, BEQUEST, HOUSING AND LIQUIDITY

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ABSTRACT

We study the impact of housing wealth and individual preferences on demand for annuities and long-term care insurance (LTCI). We build a multistate life-cycle model that includes longevity risk and health shocks. The preference is represented by a recursive utility function that separates risk aversion and elasticity of intertemporal substitution (EIS). When health shocks are considered, a higher level of risk aversion lowers the annuity demand, whereas a lower level of the EIS has the opposite effect. The impact diminishes with a weaker bequest motive, more liquid wealth, access to LTCI, or presence of home equity, all of which increase the demand for annuities. Annuity demand increases more significantly in the presence of home equity when LTCI is not offered in the market. The presence of home equity has a crowding-out effect on LTCI demand, and the effect is strengthened by a lack of bequest motives or a lower degree of risk aversion. The cash poor but asset rich may demand more LTCI coverage than their renter counterparts to preserve bequests. When both life annuities and LTCI are available, we find that the product demand is robust to changes in risk aversion and the EIS, providing insights into product designs that bundle annuities and LTCI.

Keywords: recursive utility, housing, life annuities, long-term care insurance, life-cycle model

INTRODUCTION

The occupation pension funds worldwide have undergone massive transitions from defined benefit (DB) to defined contribution (DC) schemes. DB schemes provide lifetime income streams by design, whereas DC schemes often pay out retirement benefits in capital form. Although pension funds can theoretically offer to transform capital into a lifetime payment stream to protect individuals against longevity risk, the payout phase remains largely underdeveloped with limited product offerings (Rocha, Vittas, and Rudolph 2010). The underdevelopment of the payout phase is in contrast to the accumulation phase for which fund managers have implemented multiple default investment strategies that achieve good overall performance (Duque, Morton, and Pagnoncelli 2021). It is more challenging to design the payout phase due to, among other difficulties, a high level of heterogeneity that makes it difficult to tailor products to individual needs. Retirees vary in wealth levels, homeownership status, risk tolerance, and bequest motives, all of which can affect their retirement planning. Investment strategies in the accumulation phase, by contrast, can be based solely on age, account balances, and stock market participation with minimal welfare loss (Dahlquist, Setty, and Vestman 2018).

This paper helps address the challenge pertinent to life annuities and long-term care insurance (LTCI). We focus on these two products due to their importance in tackling some of the most common retirement risks. Life annuities insure against longevity risk (i.e., the risk of outliving one's financial resources), which is the key consideration in designing the payout phase (Rocha, Vittas, and Rudolph 2011). LTCI protects against unexpected healthcare costs, which can be the single most severe spending shock for retirees

(J. Brown and Finkelstein 2011). Despite the theoretical attractiveness of the two products, both life annuity and LTCI markets appear to be underdeveloped.

We study the impact of housing wealth and individual preferences (including bequest motives and risk aversion) on demand for life annuities and LTCI. We consider longevity risk together with health shocks in a multistate life-cycle model that starts at retirement. The preference is represented by an Epstein-Zin-Weil-type utility (Epstein and Zin 1989, 1991; Weil 1989) that separately identifies risk aversion and elasticity of intertemporal substitution (EIS), which is more flexible than the commonly used power utility. We use alternative parameter values for risk aversion, the EIS, and bequest motives to investigate their impact on product demand. We assume different wealth levels and proportions of net worth in home equity to capture heterogeneous financial profiles. Although we do not intend to explain the low voluntary annuitization rate or the small private LTCI market, our results will help DC pension funds design personalized retirement products and advance the development of the payout phase.

The presence of home equity complicates the decision on life annuities and LTCI due to the size and illiquidity of home equity. This issue is relevant to the majority of older Americans; this group has high homeownership rates, hovering around 80 percent over the past few decades (US Census Bureau 2022). Homeowners who are older adults have a large fraction of household portfolios held in home equity. The median ratio of home equity to all assets is close to 60 percent in the United States (Davidoff 2009; Flavin and Yamashita 2002). The presence of home equity can pose a liquidity constraint that limits an individual's capacity to pay for insurance premiums or to support general consumption. In countries where the homeownership rates are high, retirees are often asset rich and relatively cash poor (see, e.g., Bradbury 2010; McCarthy, Mitchell, and Piggott 2002).

The impact of illiquid housing wealth on annuity demand is investigated in Pashchenko (2013), who finds that illiquid housing wealth decreases the annuity market participation rates because it reduces the amount of disposable wealth. If housing wealth provides a source of liquidity, Peijnenburg, Nijman, and Werker (2017) find a slight increase in the optimal annuitization rate. Neither paper explicitly considers the interaction between housing wealth and health shocks. In practice, housing wealth is rarely drawn on to finance nondurable consumption, and selling the house is often associated with losing spouses or moving into a nursing home (Venti and Wise 2004; Walker 2004). This means that housing wealth can be a significant source of funding for costly long-term care, thus reducing the need to keep a liquid wealth buffer. Annuity decisions can be affected by precautionary savings for health shocks (see, e.g., Davidoff, Brown, and Diamond 2005; Pang and Warshawsky 2010; Peijnenburg, Nijman, and Werker 2017; Turra and Mitchell 2008), but the ways that the liquidity released from home equity in the event of health shocks can affect annuity demand remains largely unexplored. The present study fills this gap.

Home equity can substantially reduce demand for LTCI, provided that home equity is not liquidated unless the homeowner moves to a long-term care facility. This result is proved by Davidoff (2010) in a one-period model and later confirmed by Shao, Chen, and Sherris (2019) in a multiperiod setting. Housing wealth can also reverse the complementarity between life annuities and LTCI (Davidoff 2009). A common feature of these studies is to use a power utility function that imposes an inverse relationship between risk aversion and the EIS. Since empirical experiments find no such correlation (Barsky et al. 1997), and since individuals have relative risk aversion greater than the reciprocal of the EIS (A. Brown and Kim 2013), the power utility function is unlikely to capture heterogeneous preferences of retirees.

We investigate the impact of housing wealth as well as risk aversion and EIS on demand for life annuities and LTCI by analyzing the optimal annuity and LTCI decisions for retirees of different characteristics. The choice variables include purchasing life annuities and LTCI at the point of retirement (i.e., a one-off decision) and consumption for each period while alive. Our model is limited to individuals and does not consider married couples. This is a standard approach in the literature on optimal portfolio choices for retirement. It simplifies the choice menus of retirement products and improves the model tractability. We use a Markov process to model health state transitions and fit the model to the data collected by the US Health and Retirement Study.

We focus on the female experience in that study since women are likely to live longer and so have a higher chance of requiring long-term care (Fong, Shao, and Sherris 2015). We explicitly consider the link between home equity liquidation and health shocks by assuming that entering into the state of requiring long-term care automatically triggers selling of the residence. This assumption is based on the empirical evidence that home equity is rarely spent before death unless moving into a nursing home. It also reflects our consideration for a single person rather than for a married couple since, in the event of health shocks, one-person households experience greater decline in homeownership than two-person households (Venti and Wise 2004).

We assume both life annuities and LTCI have actuarially fair prices, and abstract from product loads that are often used to explain the thin empirical demand (see, e.g., J. Brown and Finkelstein 2011; Mitchell et al. 1999), which is beyond the scope of this paper. Adding loads makes life annuities and LTCI more expensive, reducing the optimal annuitization rate and optimal LTCI coverage. Since we are interested in how demand changes with housing wealth and individual preferences rather than with the absolute value of demand, we expect no material impact of the simplified assumption on our results.

We show that housing wealth significantly enhances annuity demand when LTCI is not available. The presence of home equity can increase the optimal annuitization rate as a fraction of total wealth even though its presence reduces the proportion of total wealth that can be annuitized. Peijnenburg, Nijman, and Werker (2017) find that illiquid housing wealth slightly increases the optimal annuitization rate. We find that the result is stronger than the result in Peijnenburg, Nijman, and Werker (2017) and contrasts with Pashchenko (2013), due to our assumption of housing wealth liquidation in the event of illness. We confirm the intuitive explanation in Peijnenburg, Nijman, and Werker (2017) that home equity lowers liquid wealth buffer for health shocks, thereby increasing the amount of wealth available for annuitization. Moreover, we find that the incremental demand for life annuities due to home equity is marginal when retirees can access LTCI. As a result, the optimal annuitization rate as a proportion of total wealth decreases with housing wealth in this case.

While housing wealth can increase annuity demand, it has a crowding-out effect on LTCI (Davidoff 2010). More importantly, we reveal that housing wealth interacts with preferences to affect LTCI demand, extending the literature that considers the impact of single factors, such as housing wealth (Davidoff 2010; Shao, Chen, and Sherris 2019) or bequest motives (Lockwood 2018; Pauly 1990). We find that the crowding-out effect of home equity on LTCI demand is stronger with a weaker bequest motive or a lower degree of risk aversion. The presence of bequest motives may also reverse the crowding-out effect of home equity. We find that LTCI helps homeowners in the low wealth groups to preserve their bequests, thereby improving the demand for LTCI.

Using the recursive utility allows us to capture a wider degree of heterogeneity in preferences and pinpoint the impact of risk aversion and the EIS on product demand. When LTCI is not available, we find that a higher degree of risk aversion and a lower degree of the EIS drives the optimal annuitization rate in the opposite direction. This suggests that the power utility can confound the impact of risk aversion on annuity demand. When life annuities are not available, we find a minimal impact of EIS on LTCI demand, and also find that the result is robust to changes in homeownership status and amount of housing wealth. When both products are offered in the market, we find the demand for annuities as well as LTCI is relatively robust to changes in risk aversion and the EIS if we control for the total wealth level and the ratio of home equity to net worth. This result provides a new incentive to bundle life annuities and LTCI because it can greatly simplify the choice menus of retirement products.

The rest of the paper is organized as follows: Section 2 presents the life-cycle model and the model input in detail. Section 3 discusses how the demand for annuities and LTCI varies with wealth, homeownership, and individual preferences. Section 4 concludes.

2. LIFE-CYCLE MODEL IN RETIREMENT

This section describes the life-cycle model and its building blocks in details. We set up a discrete-time life-cycle model starting at retirement. The building blocks include the health transition model, assumptions about life annuities and LTCI, wealth dynamics, and preference representation. The life-cycle model consists of a series of one-year period that is indexed by $t \in \{1, 2, \ldots, T, T+1\}$.[1] The individual retires at $t = 1$ age 65, and her maximum attainable age is 100, so $T = 36$. All variables are defined in real terms.

2.1. HEALTH DYNAMICS AND COSTS

In each period, the retiree can be healthy, sick, or dead. We follow Ai et al. (2017), Ameriks et al. (2011), and Shao, Sherris, and Fong (2017) to consider two sick states, with one requiring long-term care and the other not. The two states vary significantly in health-care costs, and LTCI pays benefits only when one requires long-term care. We refer to the sick state that does not require long-term care as mildly disabled, and the one requiring long-term care as severely disabled. The categorization of the alive states is based on the number of difficulties in independently performing activities of daily living (ADLs). There are usually six ADLs: dressing, walking, bathing, eating, transferring, and toileting. Mildly disabled state is defined as having difficulties in one to two ADLs, and severely disabled state is defined as having difficulties in three to six ADLs. The health state at period t is denoted as s_t.

The health state transitions are modeled using a Markov process. Fong, Shao, and Sherris (2015) show that a significant proportion of older adults can recover from the disabled state to the healthy state. On the other hand, severe disability is usually chronic in nature, which substantially reduces the possibility of recovery (Ferri and Olivieri 2000; Olivieri and Pitacco 2001). We therefore allow for transition from the mildly disabled state to the healthy state and do not allow for recoveries from the severely disabled state. Figure 1 depicts the health state transitions, where 1 means healthy, 2 mildly disabled, 3 severely disabled, and 4 dead. The notation $\sigma_t(j, k)$ ($j \in \{1, 2, 3\}$, $k \in \{1, 2, 3, 4\}$) denotes the transition intensity from state j to state k at time t. A more comprehensive approach is to model both care state and health state, and so allow for all possible transitions (see, e.g., Friedberg et al. 2015). We abstract from additional features; in particular, we abstract the recovery from the state of requiring long-term care. In practice, the recovery rate from the long-term care state is very low, and insurers usually do not consider recovery when pricing LTCI, which is a prudent approach. This assumption does not contradict the finding in Friedberg et al. (2015) that older adults often move out of nursing homes. We treat living in nursing homes as one scenario of the long-term care state. Nursing home residents may move out of the facility, but they would continue to require assistance on ADLs and are still considered severely disabled.

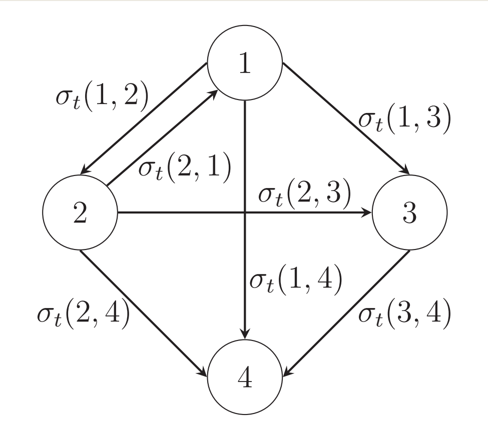

FIGURE 1. *Four-state Markov process that models health state transitions. 1, 2, 3, and 4 refer to the health states of healthy, mildly disabled, severely disabled, and dead, respectively. $\sigma_t(j, k)$ ($j \in \{1, 2, 3\}$, $k \in \{1, 2, 3, 4\}$) denotes the transition intensity from state j to state k at time t.*

Given the transition intensities, $\sigma_t(j, k)$, we can derive the n-period transition probability, denoted by $\pi^n_t(j, k) \equiv \Pr(s_{t+n} = k | s_t = j)$. The detailed derivation can be found in appendix A.

We follow Ameriks et al. (2011) to model the out-of-pocket health expenditure as a deterministic process given the health state, s_t. The deterministic process is preferred over a stochastic model (see, e.g., De Nardi, French, and Jones 2010) for its simplicity and its ability to capture the characteristics of empirical medical expense risk. Since the health-care inflation usually exceeds that of the consumer price index (CPI), it is as-

1. Note that the latest possible consumption occurs at $t = T$. The last time index $T + 1$ is for the purpose of bequest only.

sumed that the relative price of health care increases at a rate of q per annum.

2.2. HOUSING AND FINANCIAL ASSETS

The model assumes the individual lives in a mortgage-free home at retirement. Although homeowners are becoming less likely to have paid off their mortgages by retirement, data from the US Census Bureau suggests that mortgagors who are 65 and older still have lower housing costs than the general population (Channel 2021). In addition, empirical data show that housing assets are rarely drawn on unless the retiree moves to a long-term care facility (see, e.g., Venti and Wise 2004). It is assumed that the retiree will liquidate the house when she becomes severely disabled and subsequently moves to a nursing home. The house has a gross rate of return $R_{H,t+1}$ from time t to time t + 1, where $\ln(R_{H,t+1})$ follows a normal distribution with mean μ_H and variance $\sigma^2{}_H$.

The liquid assets earn a constant risk-free return of R_f . We abstract from the equity market.

2.3. RETIREMENT PRODUCTS

At retirement, the individual has access to two types of retirement products: life annuities and LTCI, both of which are offered by private companies. The retiree decides the proportion (α) of liquid assets to annuitize and the percentage coverage (λ) of LTCI to purchase. The decisions are made at retirement only.

We abstract from public offerings of life annuities (e.g., retirement benefits from social security) and LTCI (e.g., Medicaid). We assume the liquid wealth endowment includes pre-annuitized wealth.

Since public pension substitutes private life annuities (Dushi and Webb 2004), ignoring public pension leads to an upward bias in the demand for life annuities. Allowing for Medicaid will significantly complicate the model because Medicaid is means-tested and usually exempts owner-occupied property from the asset test. We instead use a consumption floor to capture some aspects of Medicaid. The details are discussed in subsection 2.4 after we introduce the consumption floor.

The life annuity is of an ordinary type that provides annual level payment for the remaining lifetime of the annuitant. The payment starts at the beginning of the first period. The annuity is charged at an actuarially fair price. Given an α proportion of liquid assets annuitized at retirement, the annual income from annuities is given by

$$\mathsf{Y} = \frac{\alpha \mathsf{B}}{1 + \sum_{t=2}^{T} R_f^{-(t-1)} \pi_1^{t-1}(s_1, s_t \neq 4)}, \tag{1}$$

where B denotes the initial endowment of liquid assets, $\pi_1^{t-1}(s_1,\ s_t\ \neq 4)$ denotes the probability that a 65-year-old individual with health state s_1 will survive for the next (t − 1) years.

LTCI covers health-care costs when the policyholder is severely disabled (i.e., health state 3). We assume a lump-sum premium and exclude any loading on the product. The actuarially fair price (P) for a full coverage LTCI policy is given by

$$\mathsf{P} = \sum_{t=2}^{T} R_f^{-(t-1)} \pi_1^{t-1}(s_1, s_t = 3) h(s_t = 3, t), \tag{2}$$

where $h(s_t,\ t)$ represents the out-of-pocket health expenditure at time t in health state st.

2.4. BUDGET CONSTRAINTS AND WEALTH DYNAMICS

In the first period, the retiree is endowed with liquid wealth of B and housing wealth of W^{H}, and the retiree is in the healthy state (i.e., health state 1). She then decides the proportion of liquid assets to annuitize and LTCI coverage to purchase. After that, she receives income from annuities (if any), incurs the health-care costs, and decides how much to consume. Let B_1 denote the amount of liquid wealth available after purchasing the retirement products. It is given by

$$B_1 = (1 - \alpha)\mathsf{B} - \lambda \mathsf{P},\ \ B_1 \geq 0. \tag{3}$$

Starting from the second period, the retiree enters the period t with health state s_t and wealth W_t, which consists of housing wealth W_t^H and liquid wealth B_t. Note that W_t, W_t^H, and B_t denote the amount available at the beginning of the period t (i.e., before any action is taken) except for B_1, which is specified in equation (3). The timing of events is as follows.

1. If $s_t = 4$, the individual is deceased, so the wealth W_t is bequeathed.
2. If $s_t < 4$, one of the following events will occur.
   a. If $s_t = 3$ and $s_{t-1} \in \{1, 2\}$, the individual will liquidate the home equity and move into a residential care facility.
   b. If $s_t = 3$ and $s_{t-1} = 3$, the individual will continue to live at the residential care facility.
   c. If $s_t < 3$, the individual will continue to live at home.
3. If $s_t < 4$, the health costs $h(s_t, t)$ are incurred and a consumption decision (C_t) is made. The remaining liquid assets earn a risk-free return R_f.

The chosen consumption level must not fall below the consumption floor C^f to ensure a minimum standard of living. If the individual's budget cannot support the minimum consumption level, we assume the government will provide subsidies to increase the consumption level to C^f. The liquid wealth in the next period is subsequently set to zero. Such a mechanism means the government is also the payer of last resort for health-care costs.

The consumption floor plays the role of Medicaid in our model, even though it is much simplified. Medicaid has a crowding-out effect on private LTCI that stems from a combined effect of means-testing and secondary payer status (J. Brown and Finkelstein 2008). Our consumption floor assumption is more stringent than means-testing and does not favor housing assets. Nevertheless, the health-care costs provided via the consumption floor mechanism have a similar secondary payer status in that the wealth available for consumption is assessed after the private policy pays the benefit.

The budget constraint for liquid assets B is given by

$$B_2 = \left(B_1 + \mathsf{Y} - h(s_1, 1) - C_1\right)^+ R_f;$$

for $t \in \{2, 3, \cdots, T\}$,

$$B_{t+1} = \begin{cases} \left(B_t + \mathsf{Y} - h(s_t, t) - C_t\right)^+ R_f & \text{if } s_t \in \{1, 2\} \\ \left(B_t + \mathsf{Y} + W_t^H \mathbb{1}_{\{s_{t-1} \in \{1,2\}\}} - (1-\lambda) h(s_t, t) - C_t\right)^+ R_f & \text{if } s_t = 3 \end{cases}, \quad (4)$$

where (·)+ is defined as max(·, 0).

The budget constraint for total wealth W is given by

$$W_2 = B_2 + W_1^H R_{H,2}, \quad \text{where } W_1^H = \mathsf{W^H};$$

for $t \in \{2, 3, \cdots, T\}$,

$$W_{t+1} = \begin{cases} B_{t+1} + W_t^H R_{H,t+1} & \text{if } s_t \in \{1, 2\} \\ B_{t+1} & \text{if } s_t = 3 \end{cases}. \quad (5)$$

2.5 PREFERENCES

Individuals in the model are assumed to have Epstein-Zin-Weil-type preferences (Epstein and Zin 1989, 1991; Weil 1989) over nonhousing consumption and a bequest. Although the housing service consumption is not directly included in the utility function, the housing wealth contributes to the utility through bequests or home equity liquidation that alleviates the budget constraint caused by excessive medical care costs.

The Epstein-Zin model generalizes the power utility model in that it can separately identify the risk aversion and the EIS. The two elements are intrinsically different. Risk aversion describes an individual's willingness to substitute consumption across different states of the world, whereas the EIS describes an individual's willingness to substitute consumption over time. When the individual's EIS is the reciprocal of the coefficient of relative risk aversion, the Epstein-Zin model reduces to the power utility model.

The preferences are specified by

$$\begin{aligned} V_t &\equiv V(B_t, W_t^H, s_t, t) \\ &= \max_{O_t}\Bigg\{(1-\beta)C_t^{1-\rho} + \beta\bigg[\mathbb{E}_t\bigg[\sum_{k\neq 4}\pi_t(s_t, s_{t+1}=k)V(B_{t+1}, W_{t+1}^H, s_{t+1}=k, t+1)^{1-\gamma} \\ &\qquad + \pi_t(s_t, s_{t+1}=4)b^\gamma W_{t+1}^{1-\gamma}\bigg]\bigg]^{\frac{1}{\theta}}\Bigg\}^{\frac{1}{1-\rho}}, \quad \theta = \frac{1-\gamma}{1-\rho}; \\ O_t &= \begin{cases} \{\lambda, \alpha, C_t\}, & \text{for } t = 1; \\ \{C_t\}, & \text{for } t = 2, \cdots, T. \end{cases} \end{aligned} \tag{6}$$

The notation V_t is the indirect utility value at time t, β the subjective discount factor, ρ the inverse of the EIS (i.e., ρ = 1/ψ), $\mathbb{E}$ the expectation operator, γ the coefficient of relative risk aversion, b the strength of bequest motive. The subjective discount factor (β) measures an individual's impatience to defer consumption. It takes values between zero and one, with a lower value representing less willingness to postpone the consumption. The strength of bequest motives (b) takes non-negative values, with a higher value meaning a stronger bequest motive.

2.6. OPTIMIZATION PROBLEM AND SOLUTION METHOD

Individuals optimize over annual consumption, annuitization rate (decided at age 65), and LTCI coverage (decided at age 65) to maximize the expected lifetime utility in (6), subject to conditions (1) to (5). We set up grid points on liquid wealth, housing wealth, and current health state to solve the optimization problem. The method of endogenous grid points (Carroll 2006) is used to set up the grid points for the liquid assets. The grid points on housing wealth are given exogenously. The log-normal distribution of house price growth is discretized by the Gauss-Hermite quadrature. The first-order condition for consumption can be solved analytically to speed up the solution process. The analytical form is derived in appendix B. The optimization problem is solved backward, starting from the last period. For the points not lying on the grid, a hybrid interpolation method introduced in Ludwig and Schön (2018) is used to find the optimal consumption and the indirect utility value.

The optimal annuitization rate and LTCI coverage are solved in the first period using the following steps. First, we set up the grid points on annuitization rate and LTCI coverage. On each grid point we solve the optimal consumption and indirect utility levels backward from the last period to the first period. Given the initial liquid wealth and housing wealth, the indirect utility value in the first period for a healthy individual can be found through the hybrid interpolation method. The optimal annuitization rate and LTCI coverage are found by searching for the grid point that gives the highest value of indirect utility.

After solving the optimal decision rules defined on the state space, the time-series profiles of a retiree's optimal consumption can be obtained through simulation. Specifically, we first simulate house price growths and health states, and then use the optimal policy rules to calculate the optimal consumption. The corresponding liquid and total wealth levels can also be obtained. The simulation is run 10,000 times

2.7. MODEL PARAMETERIZATION

We set the liquid wealth endowment at between $50,000 and $1 million, with an increment of $50,000. When retirees are endowed with home equity, we consider home equity comprises a quarter, a third, or a half of total wealth. The housing wealth proportions are lower than those reported in Davidoff (2009) and Flavin and Yamashita (2002) because the pre-annuitized wealth is implicitly included in the total wealth. The varieties of liquid wealth levels and home equity proportions allow us to investigate the impact of housing wealth and liquidity on demand for life annuities and LTCI.

We proceed to discuss the remaining inputs to the life-cycle model: health state transitions in subsubsection 2.7.1 and preference parameters in subsubsection 2.7.2.

2.7.1. HEALTH STATE TRANSITIONS

The health state transition is estimated using the data from US Health Retirement Study, which surveys a nationally representative sample of Americans over age 50 every two years, starting in 1992. The data before 1998 is

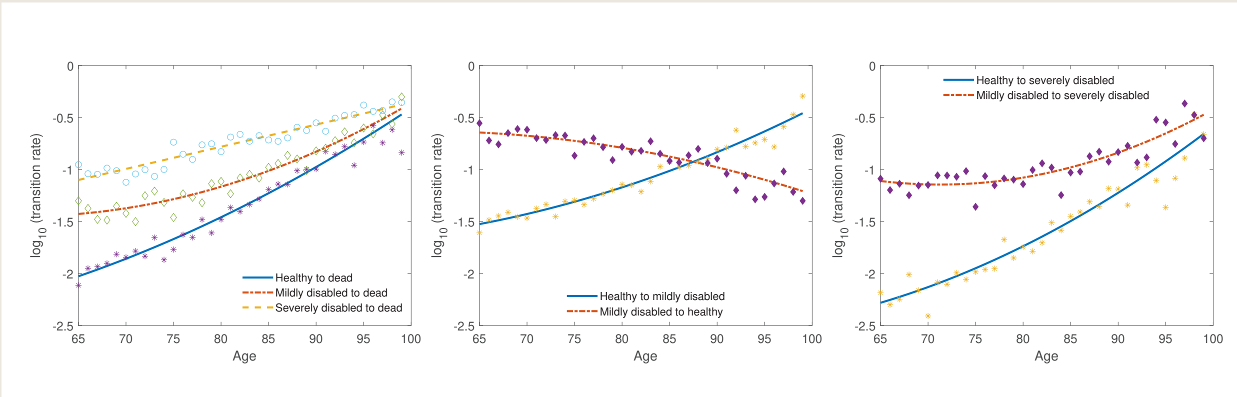

FIGURE 2. *Crude and estimated health transition rates. The scattered points are the crude rates and the curves show the estimated rates.*

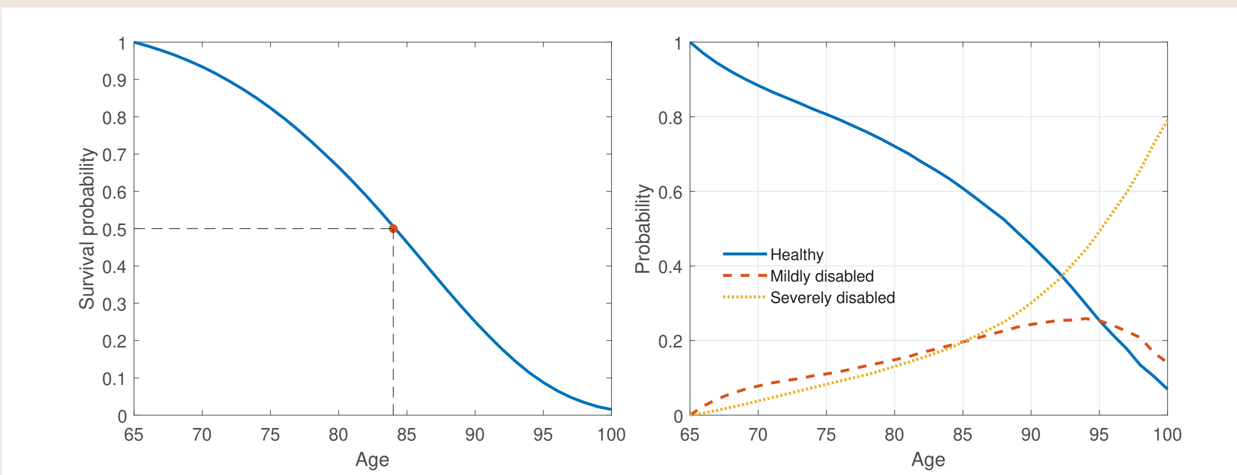

FIGURE 3. *(Left panel) Survival curve and (right panel) probability of being in each health state conditional on being alive for a 65-year-old healthy woman.*

removed due to inconsistent question structure. We use the data between 1998 and 2010 and focus on the female experiences since women have longer life expectancies than men, and so tend to spend more years in the disabled state (Fong, Shao, and Sherris 2015).

We follow the method in Fong, Shao, and Sherris (2015) to estimate the health state transitions using a generalized linear model (GLM) with the log link function. The number of transitions at age x is assumed to follow a Poisson distribution with mean (m_x) defined as a polynomial function of age.

The mean is given by

$$m_x = e_x \sum_{k=0}^{K} \eta_k x^k, \qquad (7)$$

where e_x is the central exposure to risk for x-year-old individuals, K the degree of the polynomial, η_k the coefficients of the polynomial. We use the Akaike information criterion corrected for sample size (AICc), Bayesian information criterion (BIC), and the likelihood ratio test to select the degree of polynomials. The detailed results are presented in appendix C. Figure 2 compares fitted

Parameter	Explanation	Value
Preference (Pang and Warshawsky, 2010)		
b	Strength of bequest motive	0 and 2
β	Subjective discount factor	0.96
γ	Coefficient of relative risk aversion	5
ψ	Elasticity of intertemporal substitution (EIS)	0.5
Asset returns (Yogo, 2016)		
R_f	Risk free rate	1.025
μ_H	Parameters of the lognormal distribution	0.34%
σ_H^2	of house price growth	3.5%
Consumption floor (Ameriks et al., 2011)		
C^f	Floor for healthy and mildly disabled states	$4,630
	Floor for severely disabled states	$5,640
Health expenditure (Ameriks et al., 2011)		
$h(s_1, 1)$	Initial cost for healthy state	$1,000
$h(s_2, 1)$	Initial cost for mildly disabled state	$10,000
$h(s_3, 1)$	Initial cost for severely disabled state	$50,000
$q^{\dagger}$	Health expenditure inflation in excess of CPI inflation	1.90%

† *Source*: Yogo 2016.

TABLE 1. *The parameter values used for the base case.*

transition rates with the crude rates, and shows that the estimation achieves a good fit.

We calculate the survival probability and the probability of being in each health state based on the estimated transition rates. Figure 3 shows that a 65-year-old healthy woman has a more than 50 percent chance of living to her mid-80s, and that the probability of being severely disabled increases substantially after age 85, so the overall risk of requiring long-term care is high. We follow Yogo (2016) to set the risk-free rate at 2.5 percent. As a result, the actuarially fair price of life annuities for a healthy 65-year-old individual is $14.89 per $1 of annual income, and that of LTCI is $94,752.31 for the full coverage.

2.7.2. PREFERENCE PARAMETERS

The preference parameters used in the numerical simulation take the commonly used values in the literature. Their baseline values are displayed in table 1, along with other parameter values. The sources of the parameters, unless otherwise specified, are listed in the brackets. To study the impact of bequest motives, we consider two cases: no bequest motives ($b = 0$) and a certain bequest motive ($b = 2$). We will separately change the value of γ and ψ to examine the impact of risk aversion and the EIS. The alternative values of γ are 2 and 8, and the alternative values of ψ are 0.2 and 0.7.

3. DEMAND FOR ANNUITIES AND LTCI: SCENARIO ANALYSIS

This section presents the optimal decisions on life annuities and LTCI based on the life-cycle model described in section 2. Due to the possible interaction between life annuities and LTCI (see, e.g., Ameriks et al. 2011; Koijen et al. 2016), we consider the following three scenarios: (1) annuities alone are offered (subsection 3.1), (2) LTCI alone is offered (subsection 3.2), and (3) both annuities and LTCI are offered in the market (subsection 3.3). We begin each subsection by verifying prior results in the literature before discussing the impact of housing wealth and preferences on product demand.

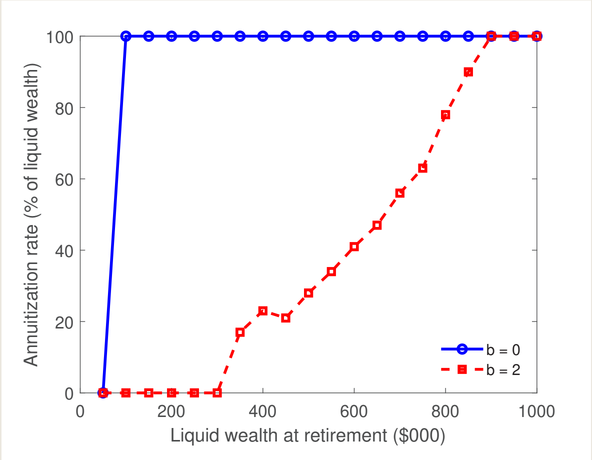

FIGURE 4. *Optimal annuitization rates for retirees endowed with liquid wealth and no housing wealth at retirement. The legend represents the strength of bequest motives. The other preference parameters are $\gamma = 5$, $\psi = 0.5$. LTCI is not offered in the market.*

3.1. ANNUITIES

Our model verifies some well-established results in the literature of optimal annuitization that abstracts from home equity. First and foremost, it has long been recognized in the literature that full annuitization is optimal for those who have no bequest motives and face no uncertainty other than their future lifetime (Davidoff, Brown, and Diamond 2005; Yaari 1965). Full annuitization remains optimal in the presence of uncertain health-care expenditures, provided that they occur later in life (Davidoff, Brown, and Diamond 2005; Peijnenburg, Nijman, and Werker 2016), while the presence of bequest motives reduces the annuity demand (Lockwood 2012). Figure 4 shows that our model reproduces the same set of results. The only exception is for those in the lowest wealth band who purchase no life annuities since they rely heavily on government transfers. In addition, figure 4 shows that higher wealth can increase the optimal annuitization rate, a result also found in Ai et al. (2017).

That individuals save from annuity income explains the optimality of full annuitization in the presence of uncertain health-care costs (Peijnenburg, Nijman, and Werker 2016). We verify this result by simulating the optimal consumption of a fully annuitized retiree endowed with $600,000 liquid wealth and no housing wealth.[2] Figure 5 shows some summary statistics of the simulated consumption paths. The mean and almost all of the quantiles are consistently below the annuity income until late in life, indicating that the annuitants save from annuity income to build up precautionary savings.

3.1.1. HOUSING WEALTH ENHANCES ANNUITY DEMAND

Having replicated the well-known results in the literature, we extend our model to include housing wealth endowment. Without bequest motives, the optimal annuitization rates are again 100 percent except for the very poor, so we henceforth focus on the case with bequest

2. The amount of $600,000 is chosen for illustrative purposes, and the same result can be found using larger or smaller amount so long as the full annuitization is optimal. We use the total wealth endowment of $600,000 in later numerical illustrations as well. The results can be extended to other wealth levels.

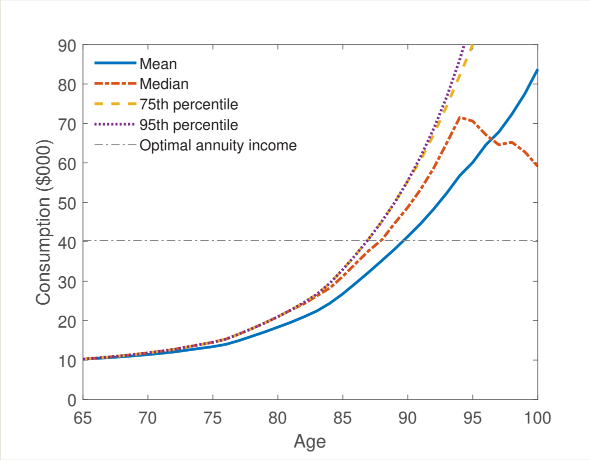

FIGURE 5. *Simulated optimal consumption for retirees endowed $600,000 liquid wealth and no housing wealth. The preference parameters are b = 0, γ = 5, ψ = 0.5. The optimal annuitization rate is 100%. LTCI is not offered in the market.*

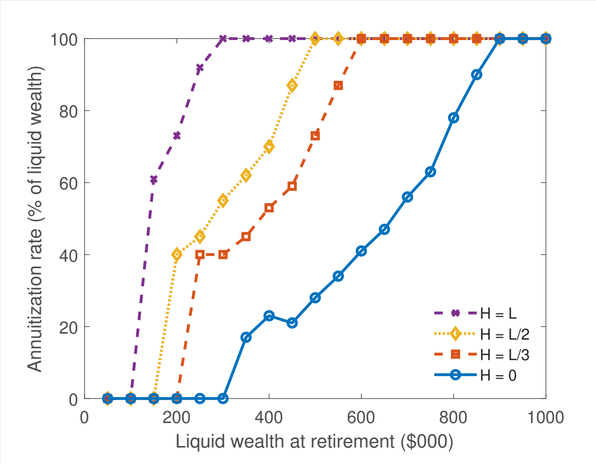

FIGURE 6. *Optimal annuitization rates for retirees endowed with liquid wealth and housing wealth. The legend represents the ratio between housing wealth (H) and liquid wealth (L) at retirement. The preference parameters are b = 2, γ = 5, ψ = 0.5. LTCI is not offered in the market.*

motives (i.e., b = 2). Due to the illiquidity of housing wealth, retirees can annuitize their liquid assets only if they are unable to access equity release products (e.g., a reverse mortgage). To assess the impact of housing wealth on annuity demand, we investigate how the amount of annuitized wealth as a proportion of liquid wealth varies with housing wealth at retirement. Figure 6 shows that, as the ratio between housing and liquid wealth grows, partial annuitization starts at a lower wealth level, and the minimum wealth required for full annuitization is also reduced. Therefore, the presence of housing wealth enhances the annuity demand.

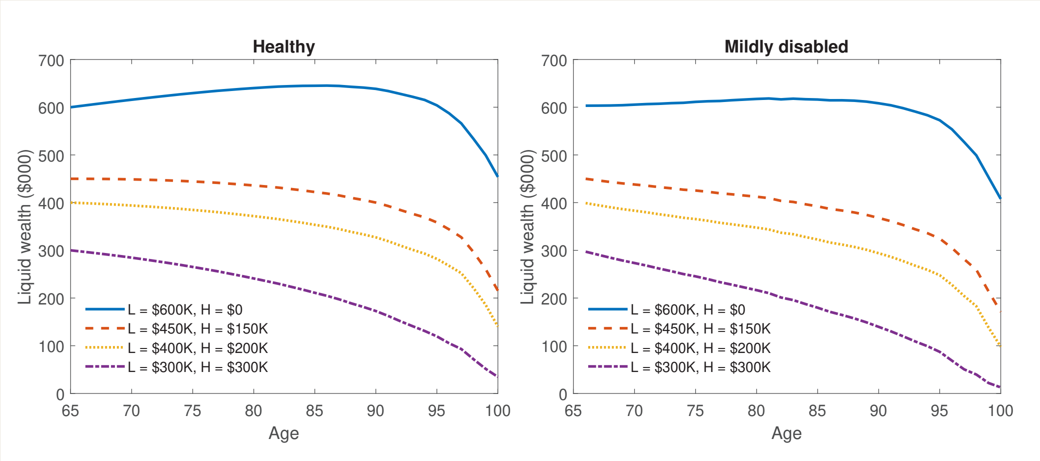

FIGURE 7. *Simulated average optimal liquid wealth paths in (left panel) healthy and (right panel) mildly disabled states. The legend represents the amount of liquid wealth (L) and housing wealth (H) at retirement. The preference parameters are b = 2, γ = 5, ψ = 0.5. Retirees do not purchase life annuities or LTCI.*

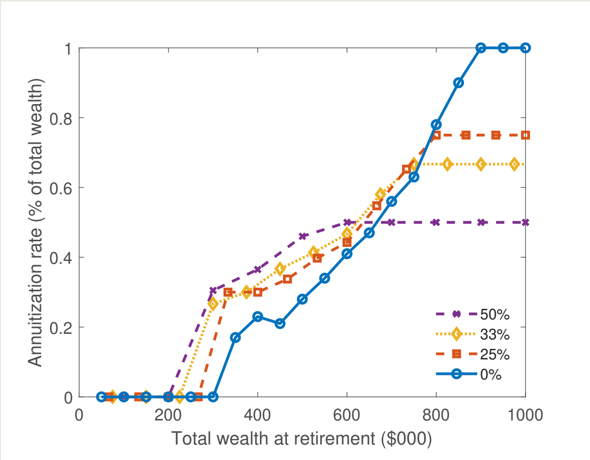

FIGURE 8. *Optimal annuitization rates (as a percentage of total wealth) for retirees with different levels of total wealth. The legend represents the proportion of total wealth in home equity at retirement. The preference parameters are b = 2, γ = 5, ψ = 0.5. LTCI is not offered in the market.*

We find that the presence of housing wealth lowers the precautionary savings from liquid wealth, thereby allowing retirees to annuitize a greater proportion of their liquid wealth. To examine the interaction between precautionary savings and housing wealth, we simulate the optimal liquid wealth paths, assuming the retiree does not purchase life annuities or LTCI. Figure 7 plots the average paths in the healthy and mildly disabled states, in which retirees hold precautionary savings.

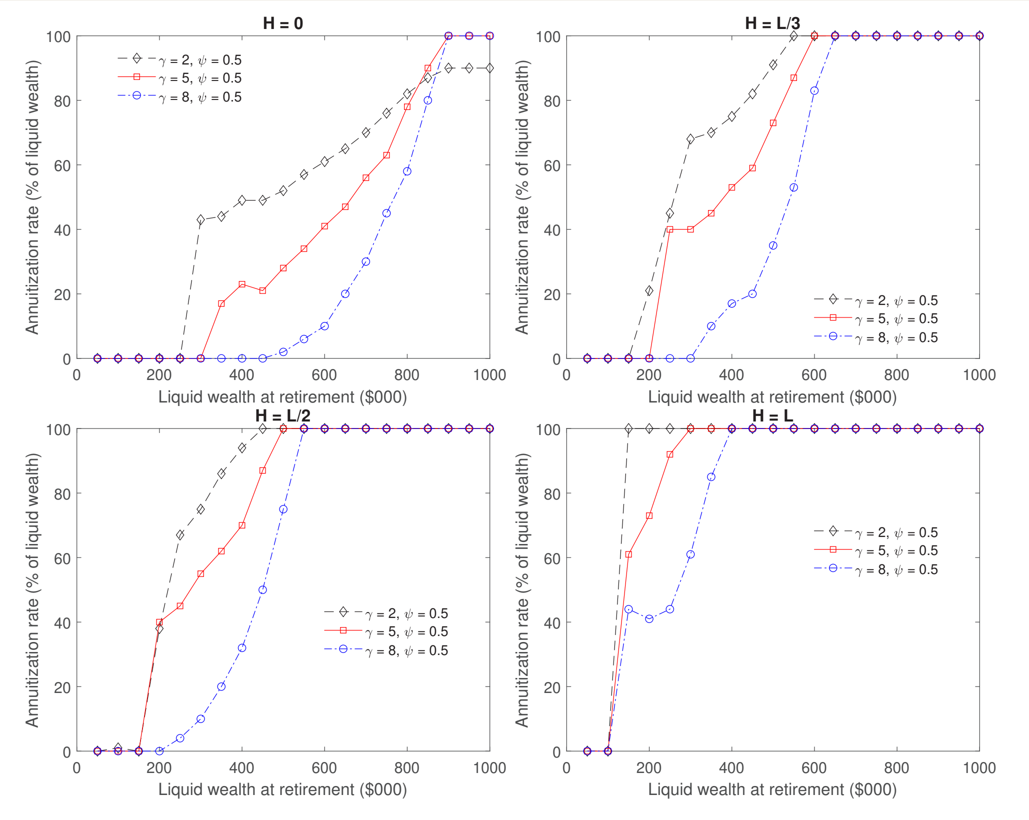

FIGURE 9. *Optimal annuitization rates for retirees with different levels of risk aversion and a certain bequest motive (b = 2). The title above each panel denotes the ratio of housing wealth (H) to liquid wealth (L) at retirement. LTCI is not offered in the market.*

When the retirement endowment has no housing component, the average liquid wealth increases slightly before declining. With a higher proportion of net worth in housing wealth, the curve first flattens and then becomes steeper. This suggests that, as housing wealth increases, retirees draw down their liquid wealth at a faster pace and use less liquid wealth relative to total wealth as precautionary savings.

While housing wealth can increase the annuity demand, its presence imposes a liquidity constraint that reduces the proportion of total wealth that can be annuitized. To investigate the net effect, in figure 8 we plot the optimal annuitization rate as a percentage of total wealth. We see that the annuitization rates are capped at the proportion of liquid wealth. Before such constraint becomes binding, the enhancement effect outweighs liquidity constraint and housing wealth increases annuity demand that is measured by the fraction of total wealth.

3.1.2. RISK AVERSION AND EIS BOTH AFFECT ANNUITY DEMAND

In addition to housing wealth, we find that both risk aversion and the EIS affect annuity demand. Figure 9 shows that a higher degree of risk aversion generally reduces the optimal annuitization rate. Individuals with

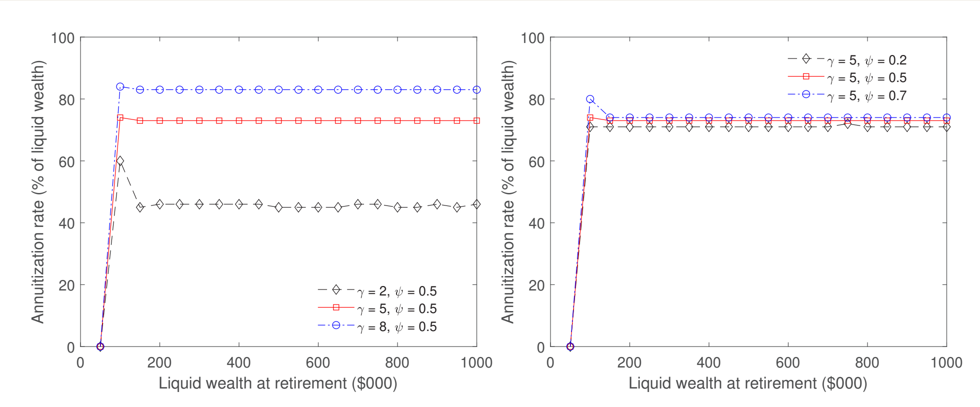

FIGURE 10. *Optimal annuitization rates for retirees with different levels of (left panel) risk aversion and (right panel) the EIS in the absence of uncertain health-care costs. The strength of bequest motives is given by b = 2. Retirees are endowed with liquid wealth and no housing wealth.*

stronger risk aversion are more averse to substituting consumption across different health states, so they set aside more liquid wealth to smooth health shocks. This in turn reduces the optimal annuitization level. Figure 9 also shows that the differences shrink with a higher level of liquid wealth or housing wealth because both factors enhance the annuity demand.

Our finding is in contrast to findings in Inkmann, Lopes, and Michaelides (2010) and Pashchenko (2013): Both documents find that the more-risk-averse retirees should purchase more annuities. Inkmann, Lopes, and Michaelides (2010) consider a different setting where a retiree can invest in the stock market and has no health-care costs. More-risk-averse individuals invest less in equities and subsequently purchase more annuities. In fact, after removing the component of health-care costs, we also find that the demand for annuities increases with risk aversion (left panel of figure 10). Pashchenko (2013) uses a power utility function where a higher degree of risk aversion is tied to a lower degree of the EIS. Our finding does not contradict hers to the extent that we find the demand for annuity generally increases with a smaller EIS, as we will discuss next.

Individuals with a higher level of the EIS are known to have higher current consumption and lower savings if the time-preference-adjusted return on savings is negative (Campbell and Viceira 1999). We replicate this result using a simplified version of our model that assumes a certain finite lifespan and no health-care costs. Furthermore, we find that a higher level of the EIS is associated with a larger amount of bequests based on the same set of assumptions. The detailed results are presented in appendix D.

After incorporating mortality risk back to the model while still abstracting from the uncertain health-care expenditure, we find that the optimal annuitization rates are similar among retirees with different levels of the EIS (right panel of figure 10). However, we find noticeable differences in the optimal consumption paths. The left panel of figure 11 shows that individuals with a higher degree of the EIS tend to have less current consumption and a flatter consumption path. Consequently, they tend to leave larger bequests (right panel of figure 11), which is consistent with our prior finding in the case of no mortality risk or health-care costs.

When facing health shocks, retirees will normally choose to hold precautionary savings if LTCI is not offered in the market. They can either annuitize less to set aside more liquid wealth up front, or save from annuity income to build up the buffer. Since our health transition model predicts that the risk of requiring long-term care increases significantly after age 85 (figure 3), retir-

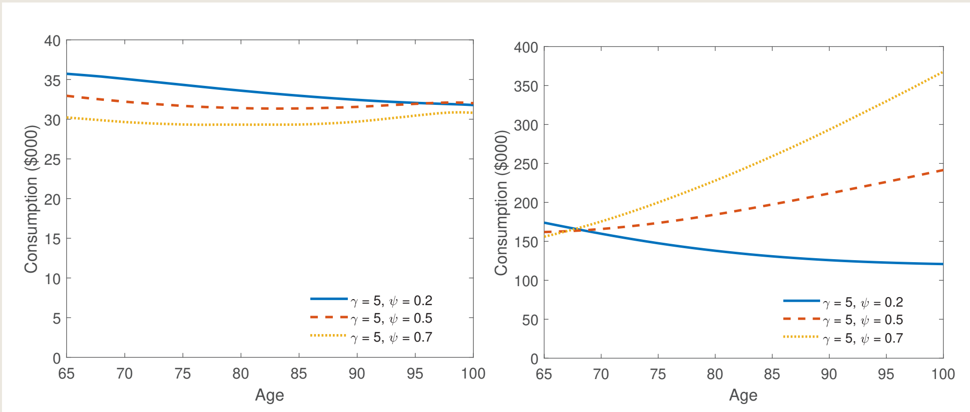

FIGURE 11. *Simulated average (left panel) optimal consumption paths and (right panel) optimal liquid wealth paths for retirees with different levels of the EIS and a certain bequest motive (b = 2). Retirees are endowed with $600,000 liquid wealth and no housing wealth at retirement. They purchase the optimal amount of annuities at retirement and have no access to LTCI.*

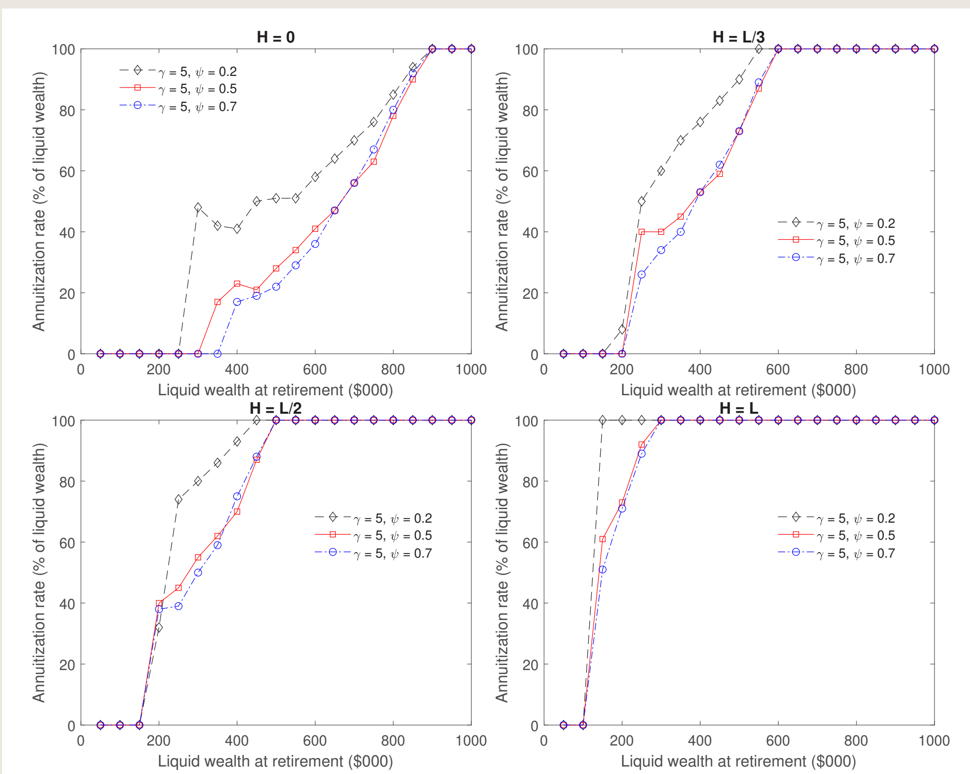

FIGURE 12. *Optimal annuitization rates for retirees with different levels of the EIS and a certain bequest motive (b = 2). The title above each panel denotes the ratio of housing wealth (H) to liquid wealth (L) at retirement. LTCI is not offered in the market.*

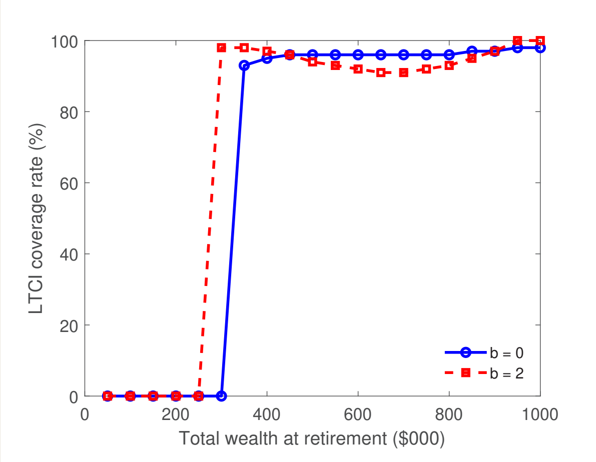

FIGURE 13. *Optimal LTCI coverage rates for retirees endowed with liquid wealth and no housing wealth at retirement. The legend represents the strength of bequest motives. The other preference parameters are $\gamma = 5$, $\psi = 0.5$. The life annuity is not offered in the market.*

ees have time to accumulate liquid wealth by spending less than the annuity income during early retirement. For someone without bequest motives, this is a more efficient strategy since wealth, if left unconsumed, generates no utility. For those with bequest motives, using a mixture of upfront savings and annuity income to build a buffer becomes optimal. Their desire to leave bequests lowers the opportunity cost of using liquid wealth as precautionary savings (Lockwood 2018). We have shown that retirees with a higher level of the EIS are likely to leave a larger amount of bequests, which implies a lower opportunity cost of holding liquid wealth. As a result, figure 12 shows that retirees with a higher degree of the EIS tend to annuitize less of their wealth. Similar to the case in figure 9, the variations in the optimal annuitization rate diminish with more liquid or housing wealth.

3.2. LTCI

LTCI is an effective instrument in managing sizable health-care costs. Figure 13 shows that retirees who are endowed with liquid wealth and no housing wealth demand nearly full LTCI coverage once their wealth levels exceed a certain threshold. Those who optimally choose to purchase no LTCI coverage rely on government transfers that provide some form of LTCI through the minimum consumption guarantee. The jump in the optimal LTCI coverage rate is not unusual. We explain that in detail in appendix E, section E.1. The impact of bequest motives is marginal, which is not surprising given the two offsetting effects of bequest motives. On the one hand, the desire to leave bequests can increase the demand for LTCI since the insurance coverage will add to the bequests left by those who die after becoming severely disabled (Pauly 1990). On the other hand, bequest motives can lower the opportunity cost of precautionary savings, thereby reducing the demand for LTCI (Lockwood 2018).

3.2.1. HOUSING WEALTH INTERACTS WITH BEQUEST MOTIVES

Figure 14 shows that more housing wealth in proportion to total wealth generally lowers the optimal LTCI coverage rate regardless of the desire to leave bequests. This is due to the substitution effect that comes from the overlap between the LTCI payment and housing wealth liquidation. A similar result is also found in Davidoff (2010) and Shao, Chen, and Sherris (2019). The comparison between the two panels in figure 14 shows that the

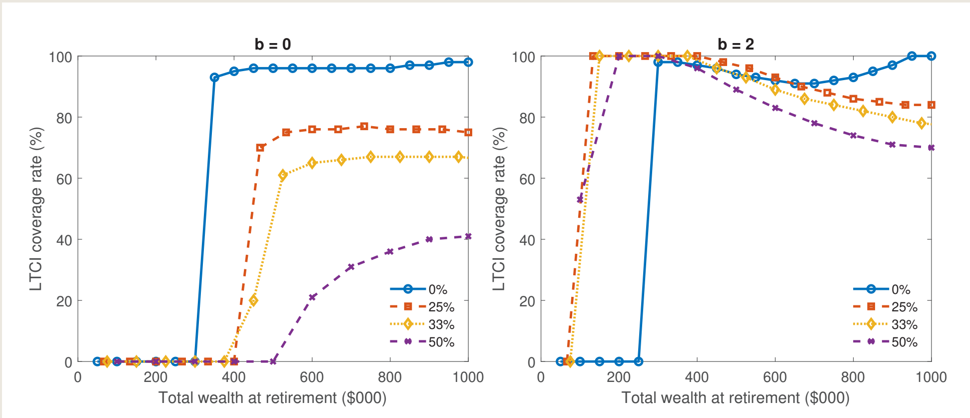

FIGURE 14. *Optimal LTCI coverage rates for retirees endowed with liquid wealth and housing wealth. The legend represents the proportion of total wealth in home equity at retirement. The preference parameters are γ = 5, ψ = 0.5. The life annuity is not offered in the market.*

FIGURE 15. *Simulated average bequests for retirees who purchase no LTCI coverage or full LTCI coverage. The title of each panel represents the amount of liquid wealth (L) and housing wealth (H) at retirement. The preference parameters are b = 2, γ = 5, ψ = 0.5. The life annuity is not offered in the market.*

gaps between the curves in the left panel are larger than those in the right panel, suggesting that bequest motives lessen the impact of housing wealth on the optimal LTCI coverage rate. This implies that, between the two offsetting effects of bequest motives, the enhancement effect dominates in the presence of home equity.

The right panel of figure 14 shows homeowners who are endowed with less than $300,000 total wealth demand far more LTCI coverage than non-homeowners who are endowed with the same amount of total wealth. This is because purchasing LTCI helps preserve bequests for homeowners more than non-homeowners in the low wealth bands, while the reduction in consumption due

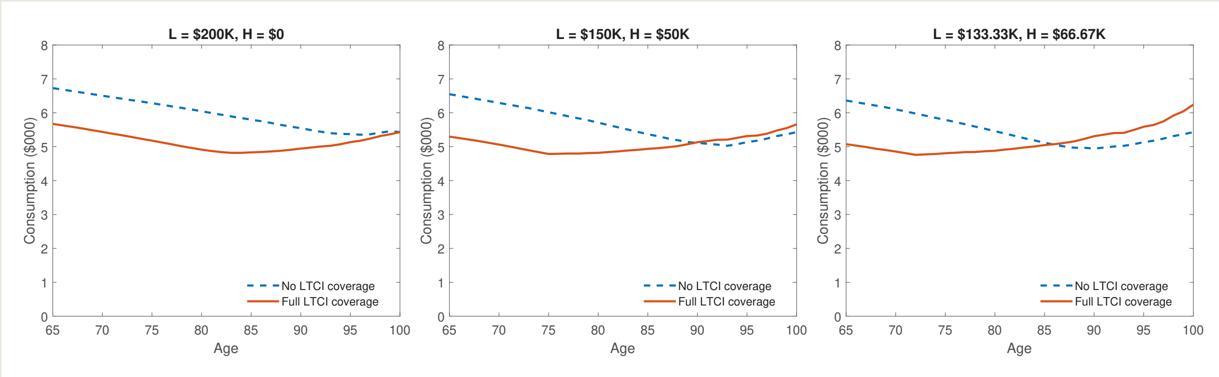

FIGURE 16. *Simulated average optimal consumption paths for retirees who purchase no LTCI coverage or full LTCI coverage. The title of each panel represents the amount of liquid wealth (L) and housing wealth (H) at retirement. The preference parameters are b = 2, γ = 5, ψ = 0.5. The life annuity is not offered in the market.*

to LTCI purchase is limited due to the minimum consumption guarantee. Figure 15 compares the average amount of bequests under no LTCI coverage and full coverage.[3] The left panel shows that the two curves are almost parallel before the solid line falls to zero, suggesting that LTCI has a limited effect in slowing the wealth drawdown for non-homeowners. By contrast, the middle and right panels of figure 15 show that increasing LTCI coverage flattens the curve for homeowners. The average amount of bequests under the full coverage almost levels off after age 85. Figure 16 shows the extent of consumption reduction caused by purchasing LTCI. The difference in annual consumption between no LTCI coverage and full coverage is, on average, around $1,000 for the first 20 years into retirement. Afterward, the gap closes due to the increased risk of requiring long-term care that incurs substantial costs and triggers the LTCI payment. For homeowners, the right two panels of figure 16 show that the average consumption with the full LTCI coverage eventually overtakes that of no LTCI coverage.

3.2.2. RISK AVERSION MORE IMPORTANT THAN THE EIS IN AFFECTING LTCI DEMAND

We find that the EIS has a minimal impact on the optimal LTCI coverage rate for retirees with and without bequest motives alike (figure 17). The same result holds for homeowners with different levels of housing wealth (see appendix E, section E.1, for more details). That the EIS has little effect on the demand for LTCI is intuitive. Unlike life annuities, which provide a constant stream of income throughout one's lifetime, LTCI provides income only when one is severely disabled, limiting its ability to smooth consumption over time. We previously argued that a higher level of the EIS strengthens the role of bequest motives in lowering the opportunity cost of liquid wealth buffers, which can reduce the demand for LTCI. The effect is offset by the enhancement made to the bequests by a higher LTCI coverage rate.

Figure 18 shows how the demand for LTCI varies with risk aversion in the absence of housing wealth. Although it appears that a higher risk aversion leads to a lower optimal LTCI coverage rate, that is not necessarily the case for homeowners, which will be discussed later. In addition, the optimal coverage rates (conditional on purchasing LTCI) in figure 18 are all close to 100 percent. Figure 19 shows that the relative difference between the optimal and the full coverage rate, in terms of the objective function, is well below 5 percent, suggesting that the utility lost from purchasing the full LTCI coverage is minimal.

3. We select the total wealth endowment amount of $200,000 for illustrative purposes. The results can be extended to other wealth levels below $300,000.

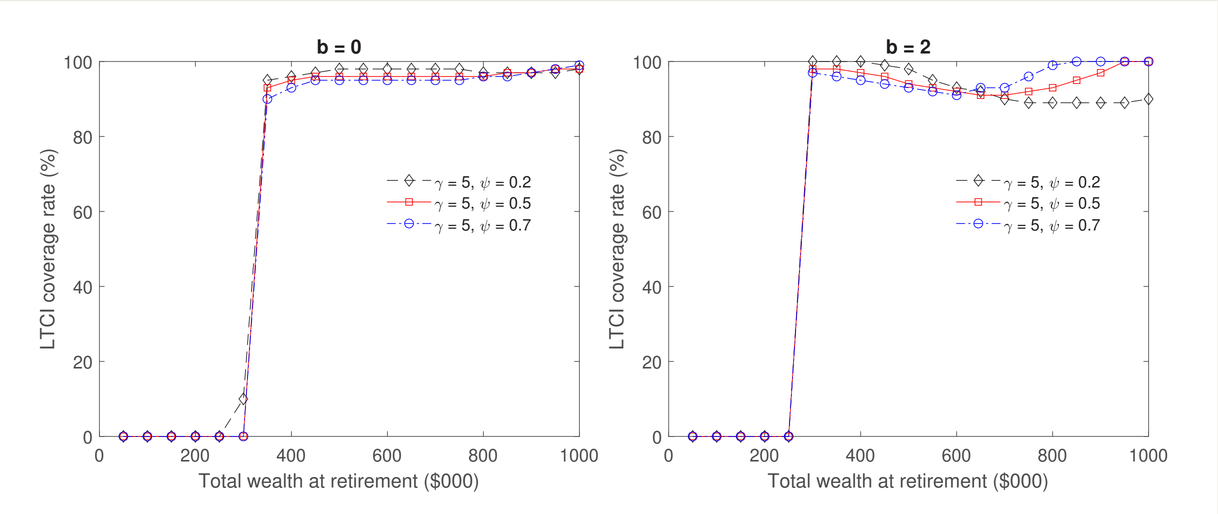

FIGURE 17. *Optimal LTCI coverage rates for retirees with different levels of the EIS. Retirees have no housing wealth. The life annuity is not offered in the market.*

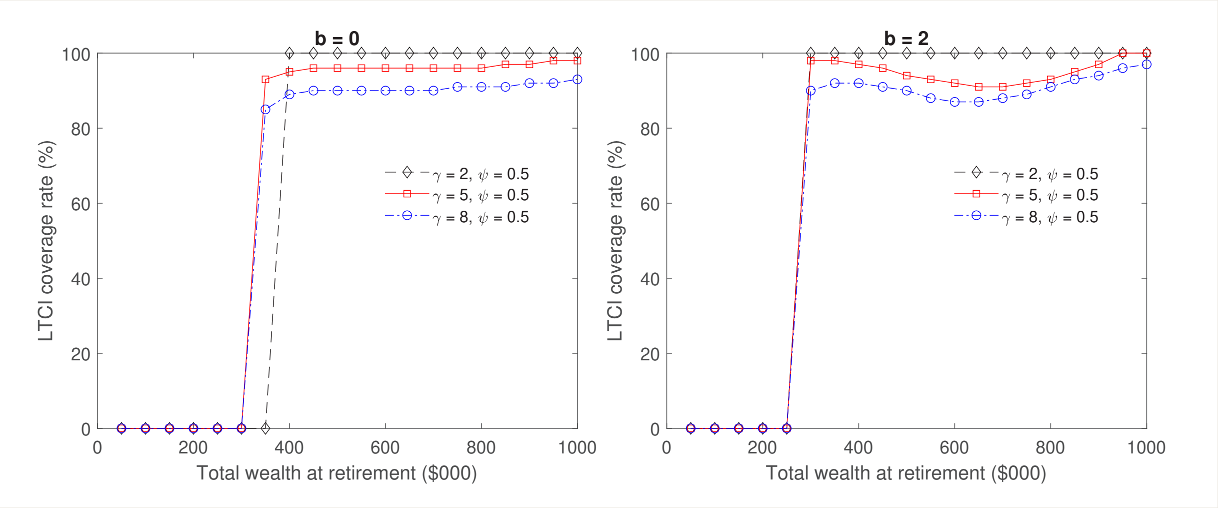

FIGURE 18. *Optimal LTCI coverage rates for retirees with different levels of risk aversion. Retirees have no housing wealth. The life annuity is not offered in the market.*

To further explain the result in figure 18 that a higher risk aversion drives down LTCI demand, we plot the simulated average optimal consumption paths in each health state, along with the overall average (figure 20). Deviating away from the optimal LTCI coverage to purchase the full amount widens the gap in consumption between the severely disabled state and other health states. Since more-risk-averse individuals prefer a smoother con-

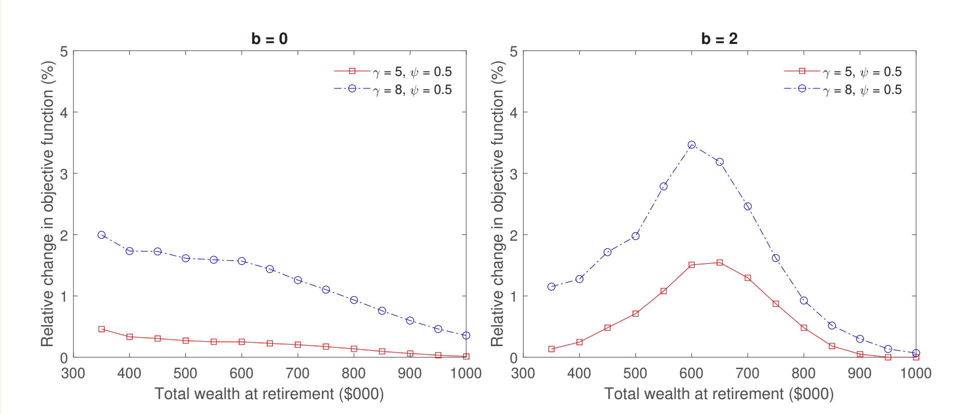

FIGURE 19. *Relative difference in the value of objective function between full LTCI coverage and optimal LTCI coverage. Retirees have no housing wealth. The life annuity is not offered in the market.*

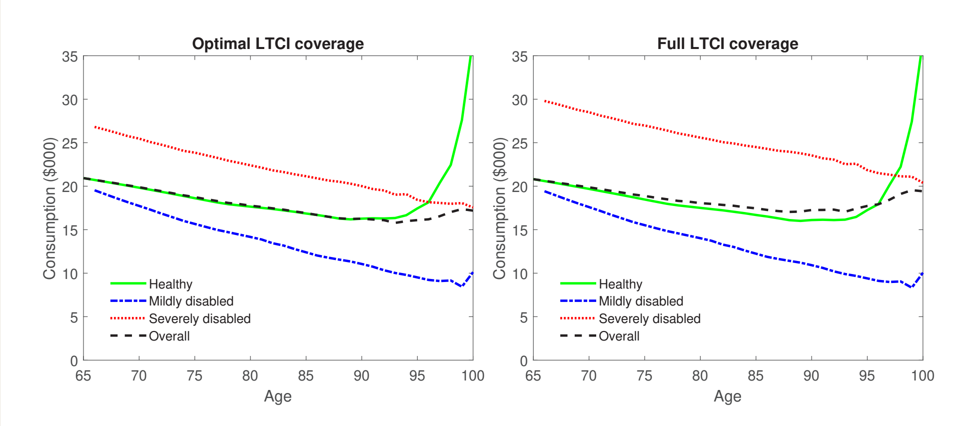

FIGURE 20. *Simulated average optimal consumption paths in each health state and the overall average. Retirees are endowed with $600,000 liquid wealth and no housing wealth at retirement. The preference parameters are $b = 0$, $\gamma = 5$, $\psi = 0.5$. The life annuity is not offered in the market.*

sumption between different health states, retirees with a relatively high level of risk aversion optimally choose to avoid the full LTCI coverage.

Furthermore, we find that risk aversion interacts with housing wealth in affecting LTCI demand. Figures 21 and 22 compare the impact of risk aversion on the optimal LTCI coverage among retirees endowed with various levels of housing wealth. As housing wealth grows, the lower the level of risk aversion, the greater the reduction in the optimal LTCI coverage rate. In one case where retirees are endowed with an equal amount of liquid and housing wealth and have no bequest motives (right panel of figure 21), the optimal LTCI coverage rate increases with risk aversion, reversing the order in figure 18.

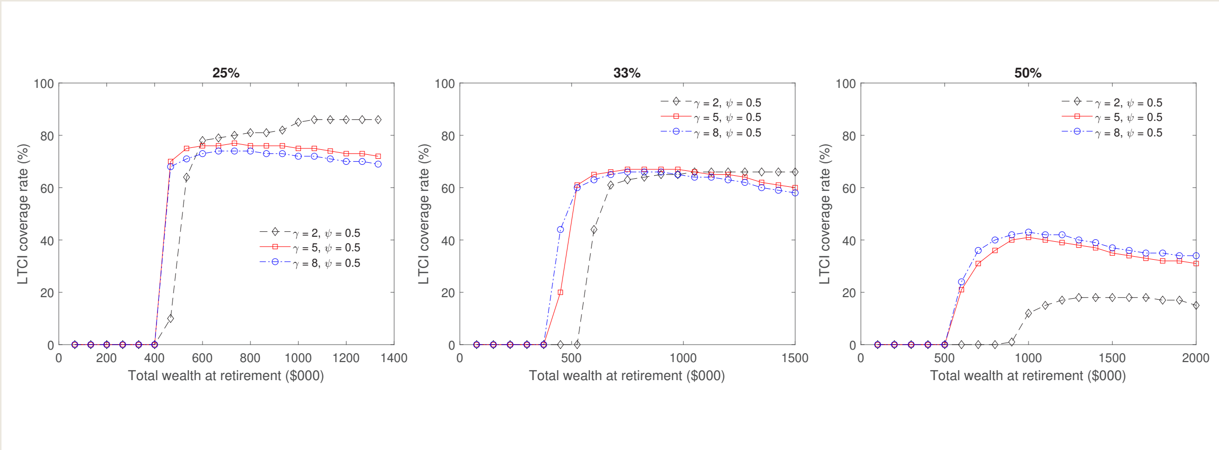

FIGURE 21. *Optimal LTCI coverage rates for retirees with different levels of risk aversion and no bequest motives. The title above each panel denotes the proportion of total wealth in home equity at retirement. The life annuity is not offered in the market.*

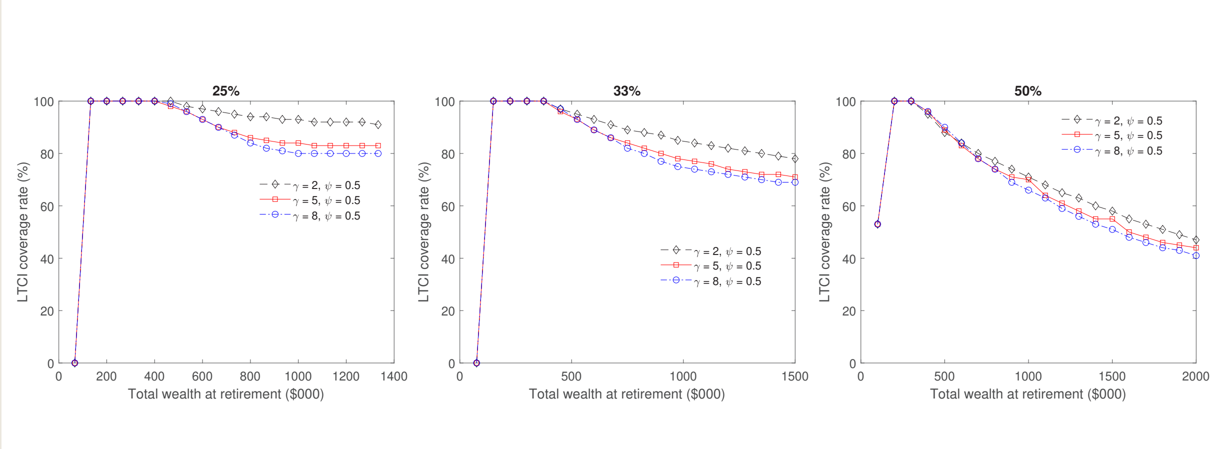

FIGURE 22. *Optimal LTCI coverage rates for retirees with different levels of risk aversion and a certain bequest motive (b = 2). The title above each panel denotes the proportion of total wealth in home equity at retirement. The life annuity is not offered in the market.*

3.3. BOTH ANNUITIES AND LTCI

LTCI is known to enhance the demand for annuities in the absence of housing wealth (see, e.g., Ameriks et al. 2008; Wu, Bateman, and Stevens 2016), and the complementarity between life annuities and LTCI can be reversed by illiquid housing wealth (Davidoff 2009). We replicate this pair of results and present the details in appendix E, section E.2.

When the life annuity or LTCI alone is offered in the market, we have shown that the product demand is affected by both housing wealth and preferences. When both products

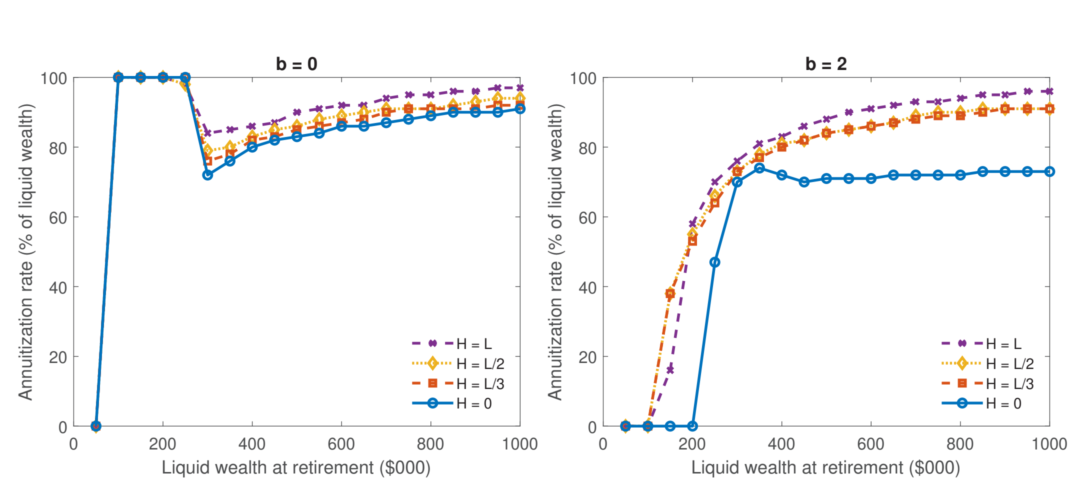

FIGURE 23. *Optimal annuitization rates (as a percentage of liquid wealth) for retirees who have access to LTCI. The legend represents the ratio between housing wealth (H) and liquid wealth (L) endowment at retirement. The preference parameters are $\gamma = 5$, $\psi = 0.5$.*

are offered, we find that the retiree's wealth level and homeownership status are more important than her risk aversion or the EIS. In addition, the bequest motives remain an important factor in determining the product demand.

3.3.1. HOUSING WEALTH AND LIQUIDITY

We have shown that housing wealth increases annuity demand for retirees with bequest motives when the life annuity alone is offered in the market. For retirees without bequest motives, the improvement is disguised by the optimality of full annuitization in the absence of housing wealth. When LTCI becomes accessible, retirees without bequest motives do not always find full annuitization optimal. Among those who partially annuitize their wealth, the optimal annuitization rates show slight improvement with housing wealth (left panel of figure 23). For retirees with bequest motives, homeownership remains an important factor in affecting the annuity demand. The right panel of figure 23 shows that homeowners tend to have higher optimal annuitization rates than non-homeowners. There is, however, little variation among homeowners endowed with the same level of liquid wealth.

Figure 24 displays the optimal annuitization rate as a proportion of total wealth. The housing wealth almost always reduces the annuity demand when retirees can access LTCI, in contrast to the case of no LTCI access (figure 8). Although housing wealth enhances annuity demand, the enhancement is unable to offset the liquidity constraint introduced by the presence of housing wealth. The reasons are twofold. First, the presence of LTCI reduces precautionary savings, thereby narrowing the gap between homeowners and non-homeowners in terms of their optimal annuitization rates as a proportion of liquid wealth (figure 23). Second, that some liquid wealth is allocated to purchase LTCI further lowers the amount of wealth that can be annuitized.

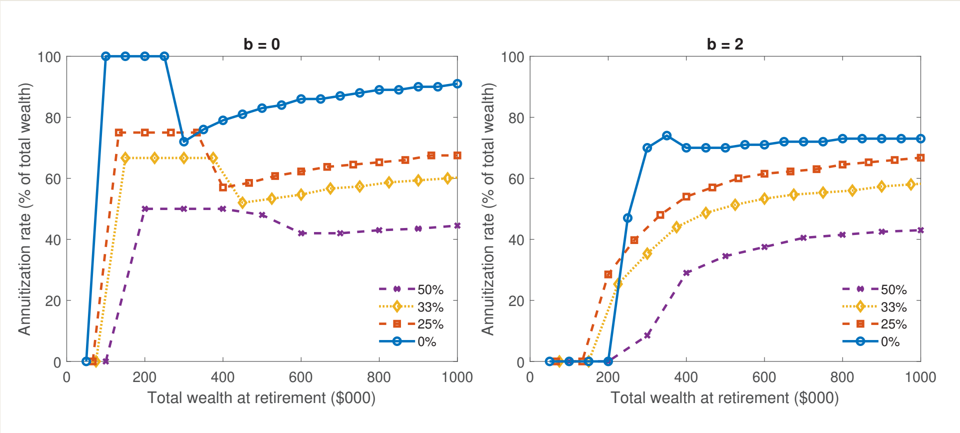

FIGURE 24. *Optimal annuitization rates (as a percentage of total wealth) for retirees who have access to LTCI. The legend represents the proportion of total wealth in housing at retirement. The preference parameters are γ = 5, ψ = 0.5. LTCI is offered in the market..*

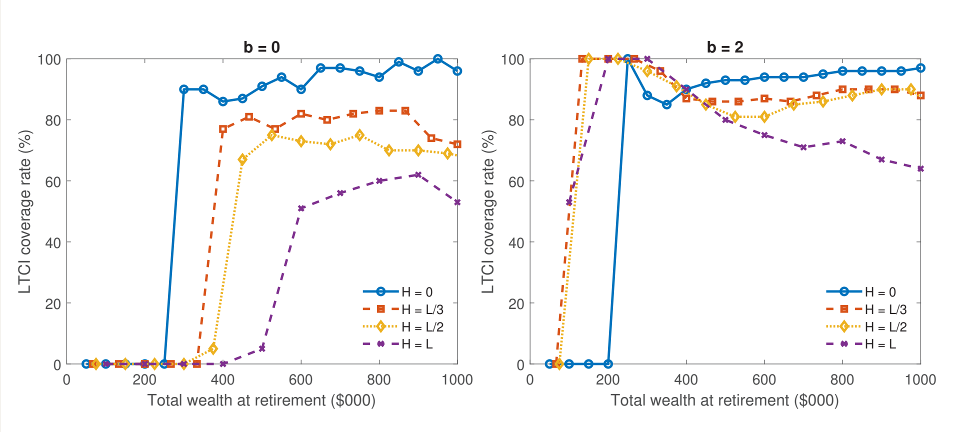

FIGURE 25. *Optimal LTCI coverage rates for retirees who have access to life annuities. The legend represents the ratio between housing wealth (H) and liquid wealth (L) endowment at retirement. The preference parameters are γ = 5, ψ = 0.5.*

Figure 25 compares the optimal LTCI coverage rate between homeowners and non-homeowners, and among homeowners with different levels of housing wealth. There are noticeable declines in the optimal LTCI coverage rates with higher housing wealth proportions, so the result that housing wealth typically weakens LTCI demand remains the same regardless of the access to life annuities. The right panel of figure 25 shows that housing wealth increases LTCI demand for retirees in the low wealth bands, similar to the case of no access to life annuities.

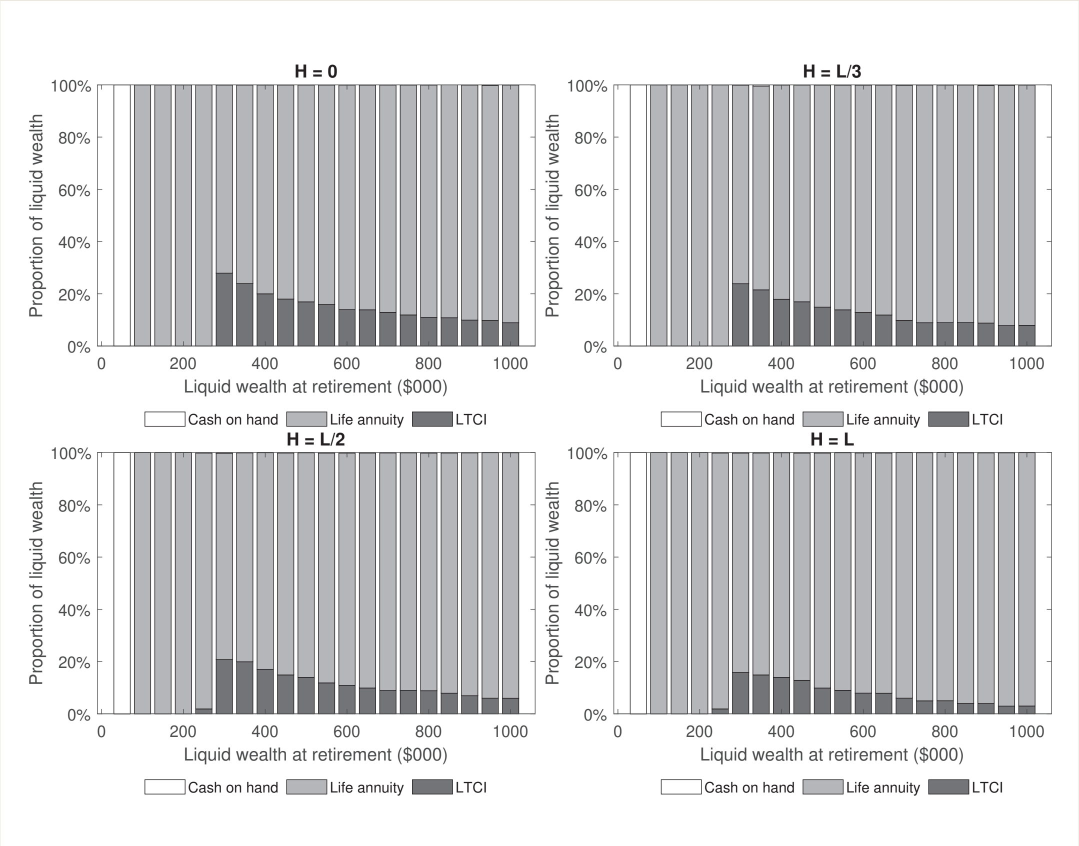

FIGURE 26. *The optimal allocation of liquid wealth endowment in the absence of bequest motives. The title above each panel denotes the ratio of housing wealth (H) to liquid wealth (L) at retirement. The preference parameters are b = 0, γ = 5, ψ = 0.5.*

Compared to figure 14, where life annuities are not offered in the market, the curves in figure 25 move more abruptly with total wealth. This is due to the result that retirees generally use all the liquid wealth to purchase the two products, which we will discuss later. Since a 1-percentage-point increase in LTCI coverage rate generally requires less liquid wealth than the same percentage point increase in annuitization rate, the capacity to purchase LTCI is more sensitive to changes in liquid wealth. While the optimal annuitization rate increases steadily with liquid wealth, the optimal LTCI coverage rate might land in a higher or lower position compared to that of the closest wealth band, depending on the budget constraint.

Figures 26 and 27 show the allocation of liquid wealth endowment in the absence and presence of bequest motives, respectively. For retirees without bequest motives, both homeowners and non-homeowners usually spend all liquid wealth on life annuities and LTCI. The proportion allocated to LTCI decreases as housing wealth grows, reflecting a weakening LTCI demand. For retirees with bequest motives, the homeownership status significantly affects the liquid wealth allocation. Homeowners generally exhaust their liquid wealth on purchasing the two products and leave little cash on hand at the point of retirement. In contrast, the top left panel of figure 27 shows that non-homeowners usually have some cash on hand after the product purchases.

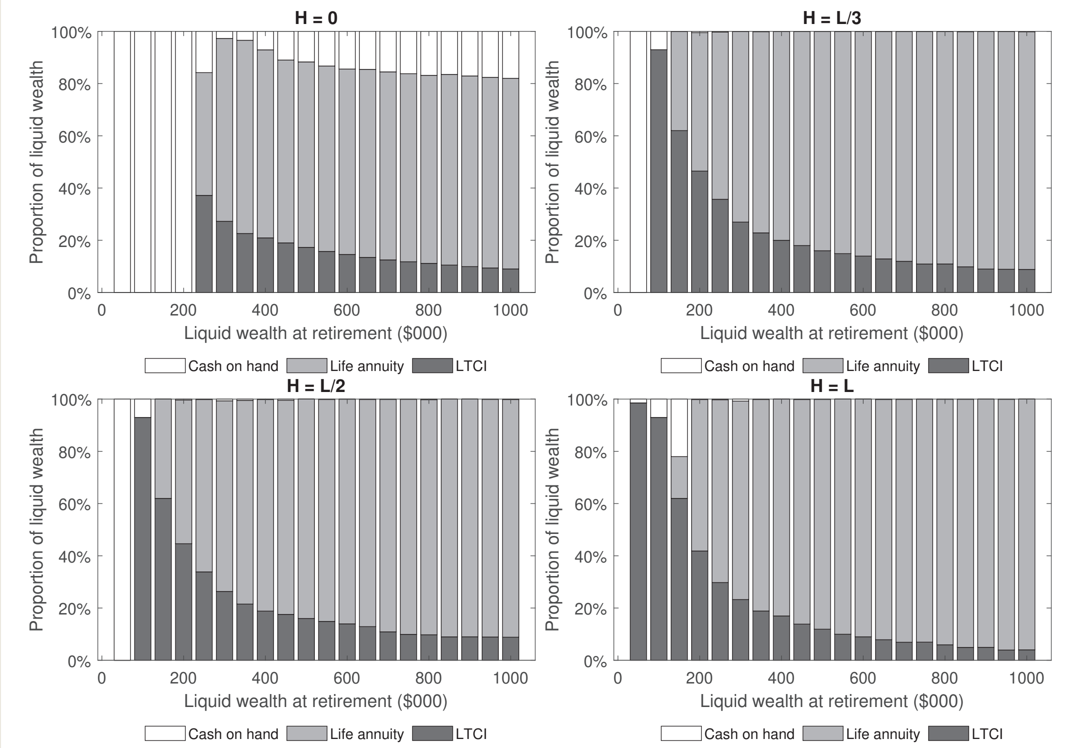

FIGURE 27. *The optimal allocation of liquid wealth endowment in the presence of bequest motives. The title above each panel denotes the ratio of housing wealth (H) to liquid wealth (L) at retirement. The preference parameters are b = 2, γ = 5, ψ = 0.5.*

For homeowners with bequest motives, the allocation to LTCI shows an exponential decay with liquid wealth. The exponential decay is mainly driven by the high LTCI coverage in the low wealth levels among homeowners (right panel of figure 25).

3.3.2. PREFERENCE

We find that risk aversion and the EIS play a far less important role in determining product demand compared to housing wealth. Figure 28 compares the optimal annuitization rate among different levels of risk aversion and the EIS in the case of no bequest motives. Figure 29 performs the same comparison for retirees with bequest motives. In both figures, the curves almost overlap. The only exception is for retirees who have a relatively low risk aversion and some bequest motives (left panel of figure 29): they show significantly less annuity demand. The comparison of the optimal LTCI coverage rate shows a similar result. In figures 30 and 31 the optimal levels vary little with the risk aversion or the EIS. We find similar results for homeowners and that the results are robust to different levels of housing wealth. The figures are displayed in appendix E, section E.2.

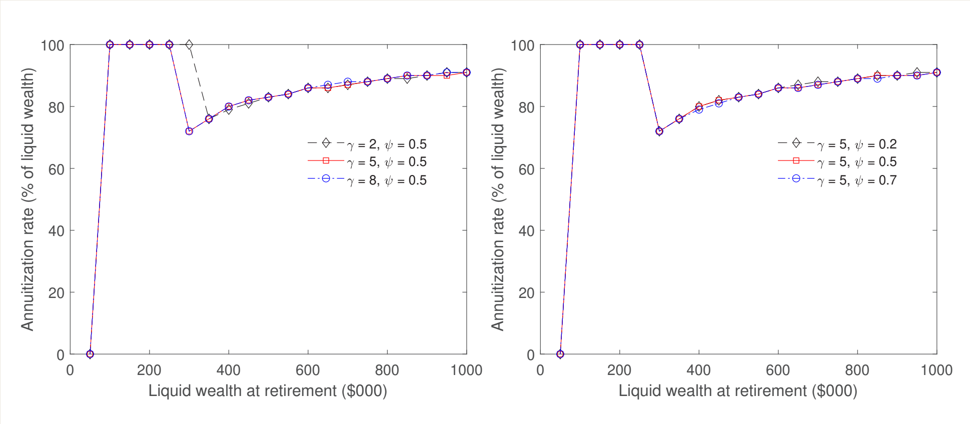

FIGURE 28. *Optimal annuitization rates for retirees with different levels of (left panel) risk aversion and (right panel) the EIS when LTCI is offered in the market. Retirees have no housing wealth and no bequest motives.*

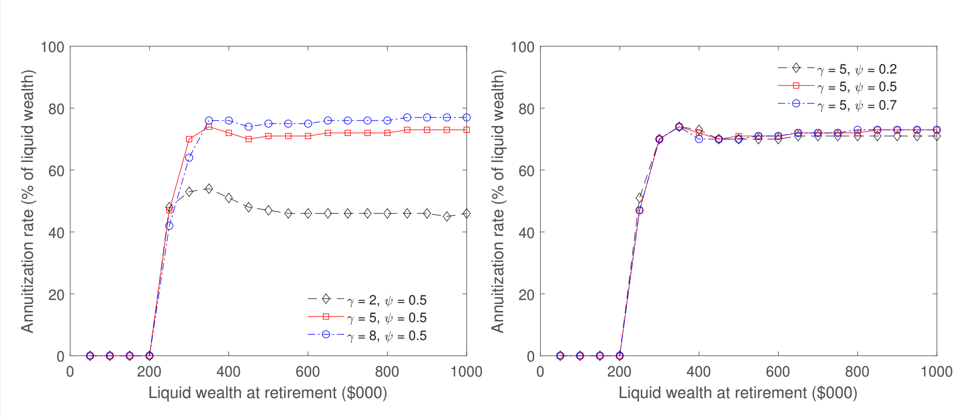

FIGURE 29. *Optimal annuitization rates for retirees with different levels of (left panel) risk aversion and (right panel) the EIS when LTCI is offered in the market. The strength of bequest motives is given by b = 2. Retirees have no housing wealth.*

In the single product case, we have shown the strong impact of bequest motives on annuity demand (figure 4) and on LTCI demand for homeowners (figure 14). When both products are available, bequest motives continue to play an important role in product decisions. Figure 23 shows that bequest motives discourage annuity purchase, especially for those in the low wealth bands or those who do not have housing wealth. Figure 25 shows a similar result to figure 14 that bequest motives improve the optimal LTCI coverage rate. Moreover, figures 26 and 27 show that bequest motives affect the liquid wealth spending, especially for non-homeowners. While retirees with no bequest motives tend to spend up their liquid wealth on the two products, non-homeowners with bequest motives leave some cash on hand.

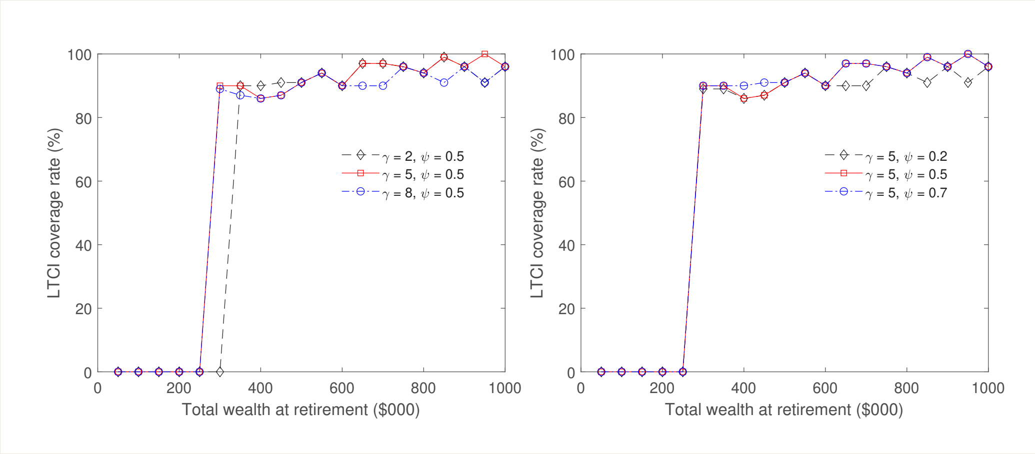

FIGURE 30. *Optimal LTCI coverage rates for retirees with different levels of (left panel) risk aversion and (right panel) the EIS when life annuities are offered in the market. Retirees have no housing wealth and no bequest motives.*

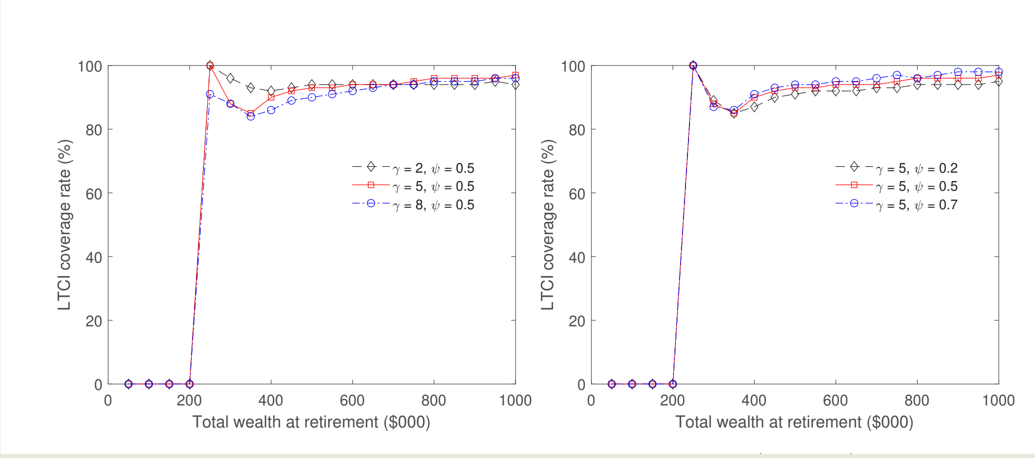

FIGURE 31. *Optimal LTCI coverage rates for retirees with different levels of (left panel) risk aversion and (right panel) the EIS when life annuities are offered in the market. The strength of bequest motives is given by b = 2. Retirees have no housing wealth.*

CONCLUSIONS

DC pension funds worldwide are reaching maturity as a growing number of members approach retirement. Those pre-retirees need to convert a lump sum into income streams to support their retirement. However, the payout phase remains less developed than the accumulation phase, exposing retirees to longevity risk and health shocks, among other risks during retirement. A major difficulty in developing the payout phase is to design personalized retirement products that meet individual needs and circumstances. Our research offers new insights to help address the challenge.

We study the impact of housing wealth and individual preferences on demand for the products that insure against longevity risk and health shocks (i.e., life annuities and LTCI). Taking into account housing wealth makes the results relevant to homeowners, who make up the majority of retirees in the United States. We use Epstein-Zin-Weil-type utility that separates risk aversion from the EIS to capture the preferences of more-heterogeneous retirees compared to the commonly used power utility function.

We find a higher level of risk aversion and a lower level of the EIS has opposite effects on annuity demand, highlighting the need to break their inverse relation imposed by the power utility function. When health shocks are considered, a higher level of risk aversion or a higher level of the EIS decreases annuity demand. The impact diminishes with weaker bequest motives, a higher level of liquid wealth, or access to LTCI, all of which enhance annuity demand. The presence of home equity enhances annuity demand, albeit to less of an extent when retirees can access LTCI.

Risk aversion and bequest motives interact with housing wealth to affect LTCI demand, while the impact of the EIS is limited. A lower degree of risk aversion strengthens the crowding-out effect of housing wealth on LTCI demand. In contrast, the crowding-out effect of housing wealth can be reduced or even reversed by bequest motives. Homeowners with limited wealth may demand higher LTCI coverage than renters who are endowed with the same amount of total wealth since LTCI can help preserve the bequests.

We find the demand for life annuities and LTCI is relatively robust to changes in risk aversion and the EIS when both products are offered simultaneously. Bequest motives, wealth levels, and homeownership status remain important factors in affecting product demand. Since information about wealth and homeownership is far easier to obtain than information about risk aversion or the EIS, the finding implies that bundling life annuities with LTCI can substantially lower the cost of designing personalized retirement products.

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ONLINE APPENDIXES

APPENDIX A. DERIVING TRANSITION PROBABILITIES

Given the transition intensities, $\sigma_t(j, k)$, the single-period transition probabilities, $\pi_t(j, k) \equiv \Pr(s_{t+1} = k|s_t = j)$, can be solved through Kolmogorov equations. In particular, we assume the transition intensities are constant within an integer age. Then the annual transition probabilities for each period are given by

$$\begin{pmatrix} \pi_t(1,1) & \pi_t(1,2) & \pi_t(1,3) & \pi_t(1,4) \\ \pi_t(2,1) & \pi_t(2,2) & \pi_t(2,3) & \pi_t(2,4) \\ \pi_t(3,1) & \pi_t(3,2) & \pi_t(3,3) & \pi_t(3,4) \\ \pi_t(4,1) & \pi_t(4,2) & \pi_t(4,3) & \pi_t(4,4) \end{pmatrix} = \exp\left[\begin{pmatrix} \sigma_t(1,1) & \sigma_t(1,2) & \sigma_t(1,3) & \sigma_t(1,4) \\ \sigma_t(2,1) & \sigma_t(2,2) & \sigma_t(2,3) & \sigma_t(2,4) \\ 0 & 0 & -\sigma_t(3,4) & \sigma_t(3,4) \\ 0 & 0 & 0 & 0 \end{pmatrix}\right],$$

where

$$\sigma_t(1,1) = -\big(\sigma_t(1,2) + \sigma_t(1,3) + \sigma_t(1,4)\big),$$

$$\sigma_t(2,2) = -\big(\sigma_t(2,1) + \sigma_t(2,3) + \sigma_t(2,4)\big),$$

$$\exp(X) = \sum_{k=0}^{\infty} \frac{1}{k!} X^k, \;\; X^0 \text{ is the identity matrix with the same dimensions as } X.$$

Given the single-period transition probabilities, the n-period transition probability, $\pi_t^n(j, k) \equiv \Pr(s_{t+n} = k|s_t = j)$, can be obtained through the Chapman-Kolmogorov equations. When $n = 1$, it reduces to the single-period transition probability (i.e., $\pi_t^1(j, k) = \pi_t(j, k)$).

APPENDIX B. FIRST-ORDER CONDITION FOR CONSUMPTION

This appendix derives the first-order condition for consumption given that LTCI coverage and annuitization decisions have been made. The method of solving optimal annuitization rate and LTCI coverage is discussed in subsection 2.6. The techniques used in this appendix build on the derivations in Munk (2013, chap. 6) who solves the optimal consumption problem for an individual with no bequest motive or health risk.

The first-order condition for C_t implies that

$$(1-\beta)C_t^{-\rho} = \beta\left\{\mathbb{E}_t\left[\sum_{k\neq 4}\pi_t(s_t,s_{t+1}=k)V_{t+1}^{1-\gamma}+\pi_t(s_t,s_{t+1}=4)b^{\gamma}W_{t+1}^{1-\gamma}\right]\right\}^{\frac{1}{\theta}-1} \times R_f\mathbb{E}_t\left[\sum_{k\neq 4}\pi_t(s_t,s_{t+1}=k)V_{t+1}^{-\gamma}\frac{\partial V_{t+1}}{\partial B_{t+1}}+\pi_t(s_t,s_{t+1}=4)b^{\gamma}W_{t+1}^{-\gamma}\right], \quad \text{(B.1)}$$

where $\partial V_{t+1}/\partial B_{t+1}$ can be derived by taking the derivative on equation (6). For the optimal decision, the equation holds without the maximum, that is

$$\begin{aligned} V_t &\equiv V(B_t, W_t^H, s_t, t) \\ &= \left\{(1-\beta)\left(C_t^*\right)^{1-\rho}+\beta\left[\mathbb{E}_t\left[\sum_{k\neq 4}\pi_t(s_t,s_{t+1}=k)V(B_{t+1}^*,W_{t+1}^H,s_{t+1}=k,t+1)^{1-\gamma}\right.\right.\right. \\ &\qquad \left.\left.\left.+\pi_t(s_t,s_{t+1}=4)b^{\gamma}\left(W_{t+1}^*\right)^{1-\gamma}\right]\right]^{\frac{1}{\theta}}\right\}^{\frac{1}{1-\rho}}, \end{aligned} \quad \text{(B.2)}$$

where C_t^* denotes the optimal consumption at time t, B_t^*+1 and W_t^*+1 denotes the next period liquid assets and total wealth, respectively, under the optimal consumption in period t. Take the derivative of equation (B.2) w.r.t. B_t

$$\begin{aligned} \frac{\partial V_t}{\partial B_t} = V_t^{\rho}\Bigg\{&(1-\beta)(C_t^*)^{-\rho}\frac{\partial C_t^*}{\partial B_t} \\ &+\beta\left[\mathbb{E}_t\left(\sum_{k\neq 4}\pi_t(s_t,s_{t+1}=k)V_{t+1}^{1-\gamma}+\pi_t(s_t,s_{t+1}=4)b^{\gamma}(W_{t+1}^*)^{1-\gamma}\right)\right]^{\frac{1}{\theta}-1} \\ &\times\mathbb{E}_t\left[\sum_{k\neq 4}\pi_t(s_t,s_{t+1}=k)V_{t+1}^{-\gamma}\frac{\partial V_{t+1}}{\partial B_{t+1}^*}\frac{\partial B_{t+1}^*}{\partial B_t}+\pi_t(s_t,s_{t+1}=4)b^{\gamma}(W_{t+1}^*)^{-\gamma}\frac{\partial W_{t+1}^*}{\partial B_t}\right]\Bigg\}, \end{aligned} \quad \text{(B.3)}$$

where $\partial B^*_{t+1}/\partial B_t$ and $\partial W^*_{t+1}/\partial B_t$ can be derived from the budget constraints (4) and (5).

$$\frac{\partial B^*_{t+1}}{\partial B_t} = \left(1 - \frac{\partial C^*_t}{\partial B_t}\right) R_f,$$
$$\frac{\partial W^*_{t+1}}{\partial B_t} = \frac{\partial W^*_{t+1}}{\partial B^*_{t+1}} \frac{\partial B^*_{t+1}}{\partial B_t} = \frac{\partial B^*_{t+1}}{\partial B_t} = \left(1 - \frac{\partial C^*_t}{\partial B_t}\right) R_f. \quad \text{(B.4)}$$

Substitute equation (B.4) into equation (B.3) and then, using the first-order condition (B.1),

$$\frac{\partial V_t}{\partial B_t} = V_t^{\rho} \beta R_f \left[\mathbb{E}_t \left(\sum_{k\neq 4} \pi_t(s_t, s_{t+1} = k) V_{t+1}^{1-\gamma} + \pi_t(s_t, s_{t+1} = 4) b^{\gamma} (W^*_{t+1})^{1-\gamma}\right)\right]^{\frac{1}{\theta}-1}$$
$$\times \mathbb{E}_t \left[\sum_{k\neq 4} \pi_t(s_t, s_{t+1} = k) V_{t+1}^{-\gamma} \frac{\partial V_{t+1}}{\partial B^*_{t+1}} + \pi_t(s_t, s_{t+1} = 4) b^{\gamma} (W^*_{t+1})^{-\gamma}\right]. \quad \text{(B.5)}$$

Consequently, the first-order condition for C_t can be rewritten as

$$\frac{\partial V_t}{\partial B_t} = (1-\beta) V_t^{\rho} C_t^{-\rho}. \quad \text{(B.6)}$$

This is the envelope condition for the preferences defined in equation (6).

Substitute the envelope condition (B.6) into equation (B.1). The first-order condition for C_t can be restated as

$$(1-\beta) C_t^{-\rho} = \beta \left\{\mathbb{E}_t \left[\sum_{k\neq 4} \pi_t(s_t, s_{t+1} = k) V_{t+1}^{1-\gamma} + \pi_t(s_t, s_{t+1} = 4) b^{\gamma} W_{t+1}^{1-\gamma}\right]\right\}^{\frac{1}{\theta}-1}$$
$$\times R_f \mathbb{E}_t \left[(1-\beta) \sum_{k\neq 4} \pi_t(s_t, s_{t+1} = k) V_{t+1}^{\rho-\gamma} C_{t+1}^{-\rho} + \pi_t(s_t, s_{t+1} = 4) b^{\gamma} W_{t+1}^{-\gamma}\right]. \quad \text{(B.7)}$$

Therefore, the optimal consumption in period t is given by

$$C^*_t = \left\{\beta R_f \left\{\mathbb{E}_t \left[\sum_{k\neq 4} \pi_t(s_t, s_{t+1} = k) V_{t+1}^{1-\gamma} + \pi_t(s_t, s_{t+1} = 4) b^{\gamma} W_{t+1}^{1-\gamma}\right]\right\}^{\frac{1}{\theta}-1}\right.$$
$$\left.\times \mathbb{E}_t \left[\sum_{k\neq 4} \pi_t(s_t, s_{t+1} = k) V_{t+1}^{\rho-\gamma} C_{t+1}^{-\rho} + \pi_t(s_t, s_{t+1} = 4) \frac{b^{\gamma}}{1-\beta} W_{t+1}^{-\gamma}\right]\right\}^{-\frac{1}{\rho}}. \quad \text{(B.8)}$$

In the terminal period, $\pi_T(s_T, s_{T+1} = k) = 0$ for $k \in \{1, 2, 3\}$ and $\pi_T(s_T, s_{T+1} = 4) = 1$, so the optimal consumption in period T becomes

$$C_T^* = \left(\frac{1}{\beta} - 1\right)^{\frac{1}{\rho}} \left\{\mathbb{E}_T\left[b^\gamma W_{T+1}^{1-\gamma}\right]\right\}^{\frac{1}{\rho} - \frac{1}{\rho\theta}} \times \left\{R_f \mathbb{E}_T\left[b^\gamma W_{T+1}^{-\gamma}\right]\right\}^{-\frac{1}{\rho}}. \quad \text{(B.9)}$$

APPENDIX C. SUPPLEMENTARY RESULTS OF HEALTH STATE TRANSITIONS

Tables C.1 and C.2 show the number of transitions and exposure years, respectively, in five-year intervals. The results are used to calculate the crude transition rates, which will then be graduated using the generalized linear model. Table C.3 displays the results of selecting the appropriate degree of the polynomial in equation (7).

	$1 \to 2$	$1 \to 3$	$1 \to 4$	$2 \to 1$	$2 \to 3$	$2 \to 4$	$3 \to 4$
50 – 54	67	21	8	52	13	2	4
55 – 59	280	40	55	212	69	27	16
60 – 64	458	74	114	436	129	37	36
65 – 69	553	112	193	474	147	86	79
70 – 74	575	107	226	441	178	97	86
75 – 79	579	144	257	349	157	116	171
80 – 84	570	162	315	338	190	166	242
85 – 89	445	172	302	235	211	212	312
90 – 94	218	92	160	86	156	172	296
95 – 100	52	24	51	18	76	75	174
Total	3,797	948	1,681	2,641	1,326	990	1,416

Note: '1' is healthy state, '2' mildly disabled state, '3' severely disabled state, '4' dead state.

TABLE C.1. *Number of Transitions between Different Health States.*

	Healthy	Mildly disabled	Severely disabled
50 – 54	4,527.18	361.92	121.51
55 – 59	10,816.97	1,136.76	387.61
60 – 64	15,721.89	1,811.16	692.93
65 – 69	16,610.65	2,146.23	802.31
70 – 74	13,975.53	2,079.22	948.19
75 – 79	10,807.98	2,164.77	1,071.76
80 – 84	7,512.86	2,131.81	1,242.44
85 – 89	3,870.87	1,826.11	1,457.01
90 – 94	1,235.42	965.27	1,006.33
95 – 100	235.92	265.35	421.37
Total	85,315.27	14,888.60	8,151.45

TABLE C.2. *Number of Exposure Years in Healthy, Mildly Disabled, and Severely Disabled States.*

APPENDIX D. ROLE OF EIS ON CONSUMPTION AND SAVINGS

Figures D.1 and D.2 replicate the well-established results in the literature using a simplified version of our model that removed mortality risk and health-care costs. Retirees are assumed to have a certain lifespan and will die upon reaching the age of 100 years old. Figure D.1 compares the simulated average consumption paths for different levels of the EIS. Individuals with a higher level of the EIS tend to have more-current consumption and a steeper consumption path over time. Since the time-preference-adjusted return on savings is negative in our model (i.e., $R_f - 1 < 1 - \beta$), the result is consistent with the finding in Campbell and Viceira (1999).

Figure D.2 shows the simulated average liquid wealth paths for different levels of the EIS. The left panel shows that, the more the EIS, the lower the curve. In other words, more-current consumption leads to less savings in the absence of bequest motives, in line with the result in Campbell and Viceira (1999). The right panel of figure D.2 shows that this result can be reversed after introducing the bequest motives. Since retirees have a certain lifespan, the amount of bequests is the same as the terminal wealth. We find that a higher level of the EIS is associated with greater bequests.

K	AICc	BIC	D_c	ΔD_c
Disability				
$\sigma_t(1, 2)$: healthy to mildly disabled				
1	334.84	337.96	87.51	
2	304.56	**309.05**	**54.90**	**32.62*** **
3	**303.87**	309.61	51.74	3.16*
$\sigma_t(1, 3)$: healthy to severely disabled				
1	260.49	263.60	64.61	
2	247.74	**252.23**	**49.53**	**15.08*** **
3	**246.66**	252.40	45.99	3.54*
$\sigma_t(2, 3)$: mildly disabled to severely disabled				
1	316.44	319.55	100.70	
2	279.25	**283.74**	**61.17**	**39.52*** **
3	**279.14**	284.88	58.60	2.57
Recovery				
$\sigma_t(2, 1)$: mildly disabled to healthy				
1	301.16	304.27	73.30	
2	**292.57**	**297.06**	**62.38**	**10.92*** **
3	294.97	300.72	62.32	0.06
Mortality				
$\sigma_t(1, 4)$: healthy to dead				
1	272.53	275.64	51.01	
2	**265.01**	**269.50**	**41.16**	**9.85*** **
3	267.02	272.77	40.71	0.45
$\sigma_t(2, 4)$: mildly disabled to dead				
1	246.79	249.90	45.02	
2	**243.68**	**248.18**	**39.58**	**5.44** **
3	244.11	249.85	37.54	2.04
$\sigma_t(3, 4)$: severely disabled to dead				
1	**245.02**	**248.13**	**29.59**	
2	247.35	251.85	29.58	0.00
3	247.45	253.20	27.22	2.36

Note: The chosen degree of polynomial value is in **bold** for each set of nested models. D_c is the residual deviance statistics. ΔD_c denotes the test statistics for the likelihood ratio test. * is for statistic that is significant at the 10% level, ** at the 5% level, *** at the 1% level.

TABLE C.3. *Model Selection of the Poisson Generalized Linear Models.*

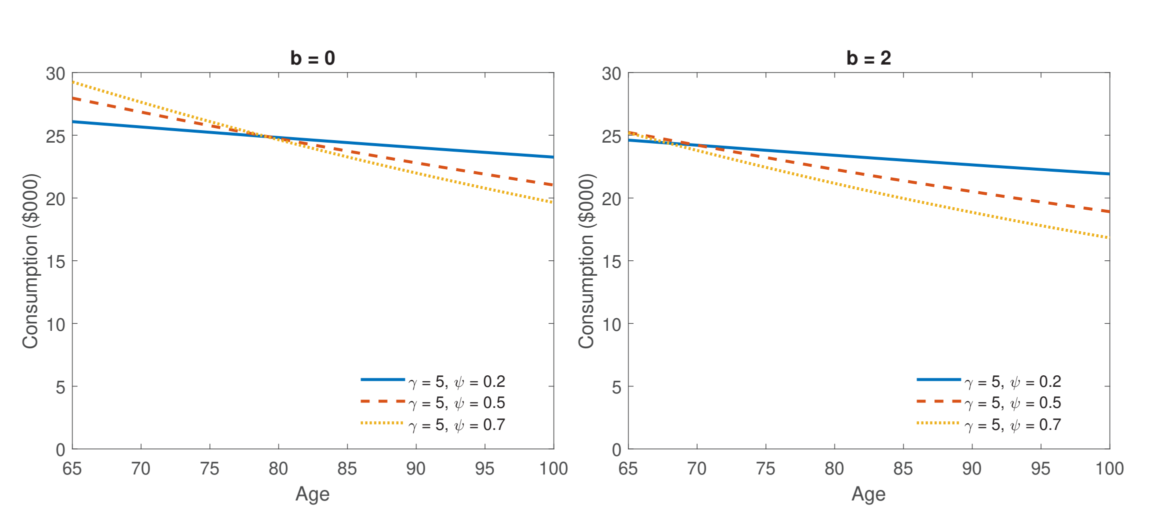

FIGURE D1. *Simulated average consumption paths for retirees with different levels of the EIS. The 65- year-old retirees have a certain lifespan of 35 years and face no health-care costs. They are endowed with $600,000 liquid wealth and no housing wealth at retirement.*

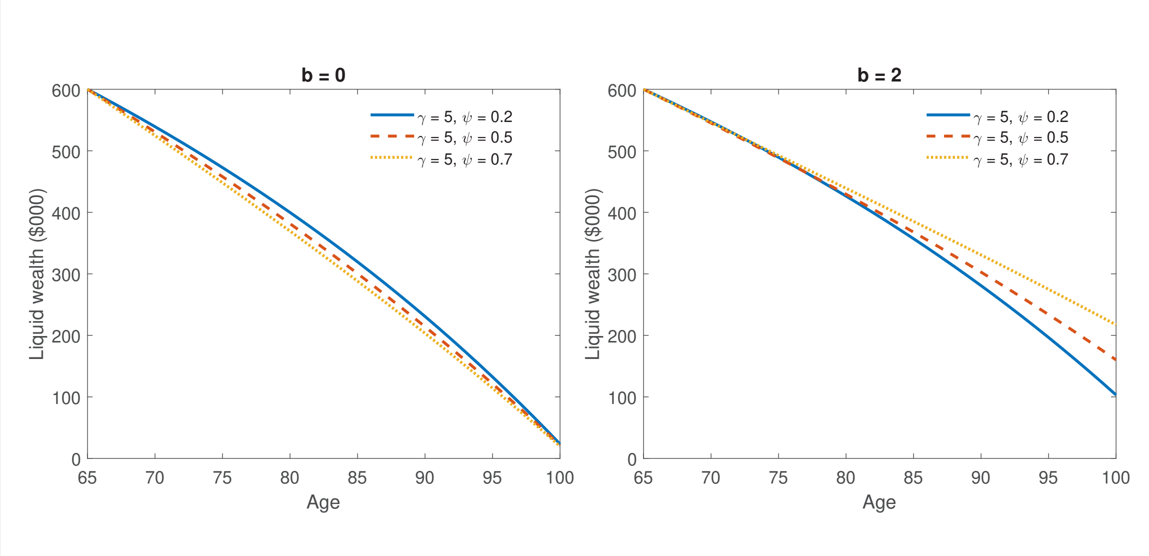

FIGURE D2. *Simulated average liquid wealth paths for retirees with different levels of the EIS. The 65-year-old retirees have a certain lifespan of 35 years and face no health-care costs. They are endowed with $600,000 liquid wealth and no housing wealth at retirement.*

APPENDIX E. SUPPLEMENTARY RESULTS OF PRODUCT DEMAND

E.1. LTCI

The jump in the optimal LTCI coverage rate shown in figure 13 is not unusual. Shao, Chen, and Sherris (2019) also find strong demand for LTCI at different wealth levels, although they set the lowest wealth level at $240,000, below which retirees are likely to have a minimal demand due to government transfers. The jump can be explained by the nonlinear effect of LTCI on consumption. Figure E.1 shows the average optimal consumption paths in the severely disabled state under different LTCI coverage rates. Regardless of bequest motives, the increment in the optimal consumption grows considerably when LTCI coverage increases in equal steps from 0 percent to 100 percent. By contrast, the average consumption in the healthy and mildly disabled states changes more or less evenly as LTCI coverage increases in equal steps (figures E.2 and E.3). The nonlinear effect on consumption implies that, if one does not completely rely on government transfers and purchases some LTCI coverage, the marginal benefit of an extra coverage rate can easily exceed its marginal cost when the coverage is not high. Therefore, the optimal coverage rate for non-homeowners is either at the high end or the low end.

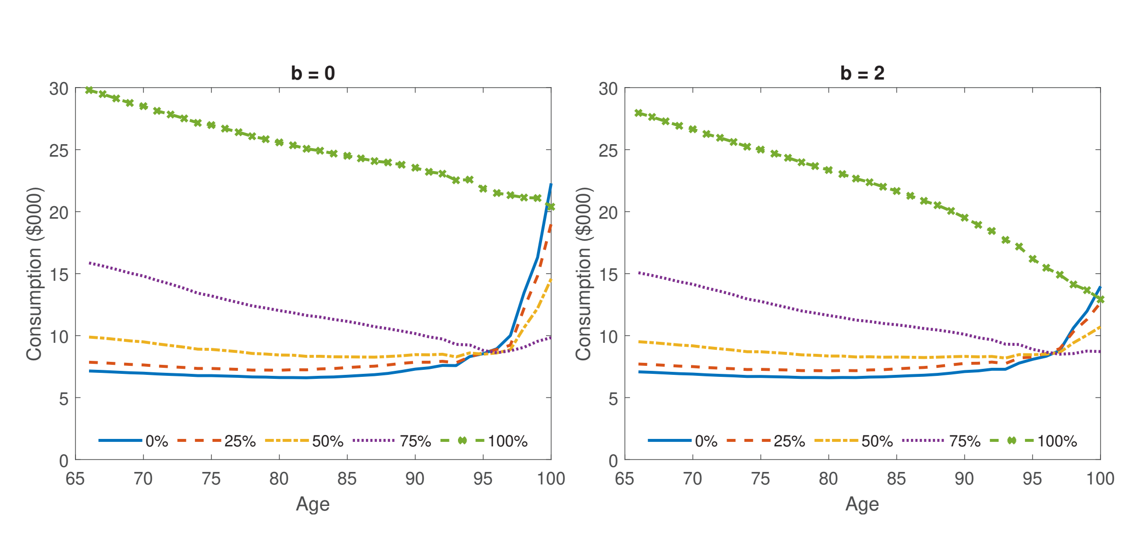

FIGURE E1. *Simulated average optimal consumption paths in the severely disabled state. The legend represents different LTCI coverage rates. Retirees are endowed with $600,000 liquid wealth and no housing wealth at retirement. The preference parameters are $\gamma = 5$, $\psi = 0.5$. The life annuity is not offered in the market.*

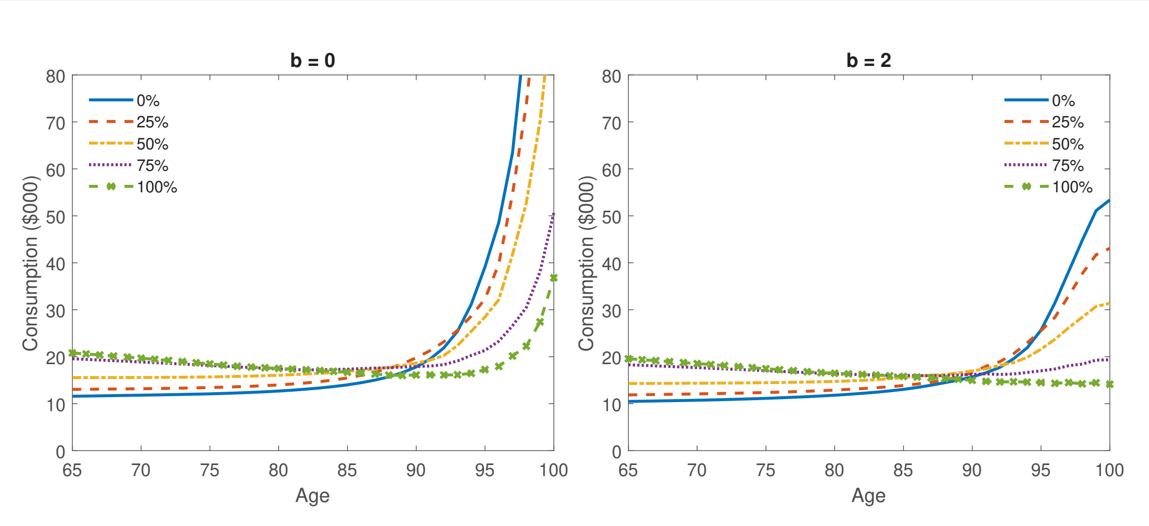

FIGURE E2. *Simulated average optimal consumption paths in the healthy state. The legend represents different LTCI coverage rates. Retirees are endowed with $600,000 liquid wealth and no housing wealth at retirement. The preference parameters are $\gamma = 5$, $\psi = 0.5$. The life annuity is not offered in the market.*

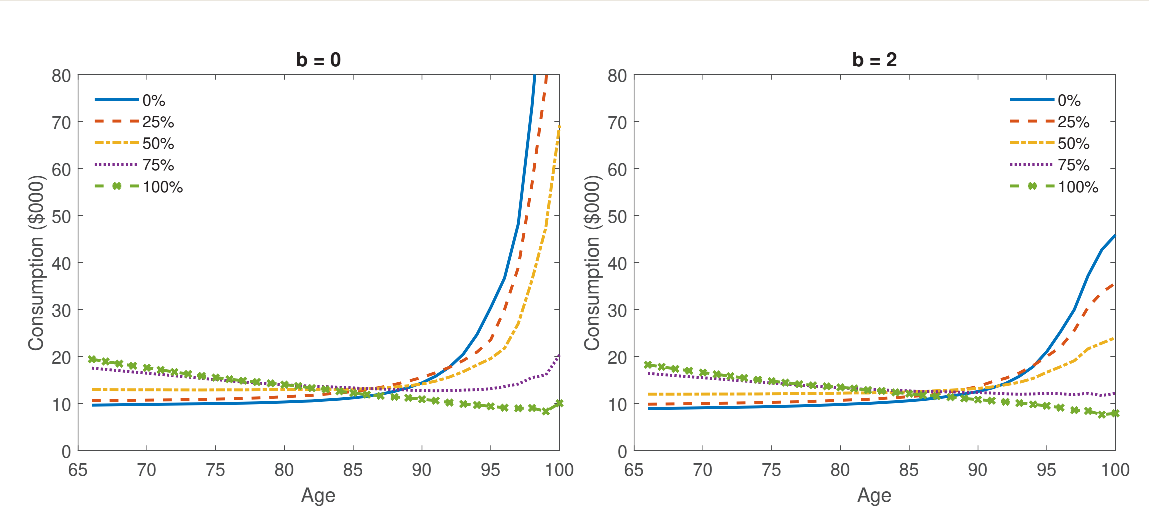

FIGURE E3. *Simulated average different consumption paths in the mildly disabled state. The legend represents different LTCI coverage rates. Retirees are endowed with $600,000 liquid wealth and no housing wealth at retirement. The preference parameters are $\gamma = 5$, $\psi = 0.5$. The life annuity is not offered in the market.*

Figures E.4 and E.5 supplement figure 17, which shows that the EIS has a minimal impact on the optimal LTCI coverage rate for non-homeowners. Figure E.4 displays the optimal LTCI coverage rates with different levels of the EIS for homeowners who do not have bequest motives, each panel representing a different housing wealth proportion. Figure E.5 shows the same comparison for homeowners with bequest motives. Overall, the EIS has a minimal impact on the optimal LTCI coverage rate for homeowners with and without bequest motives alike, and this result is robust to varying levels of housing wealth.

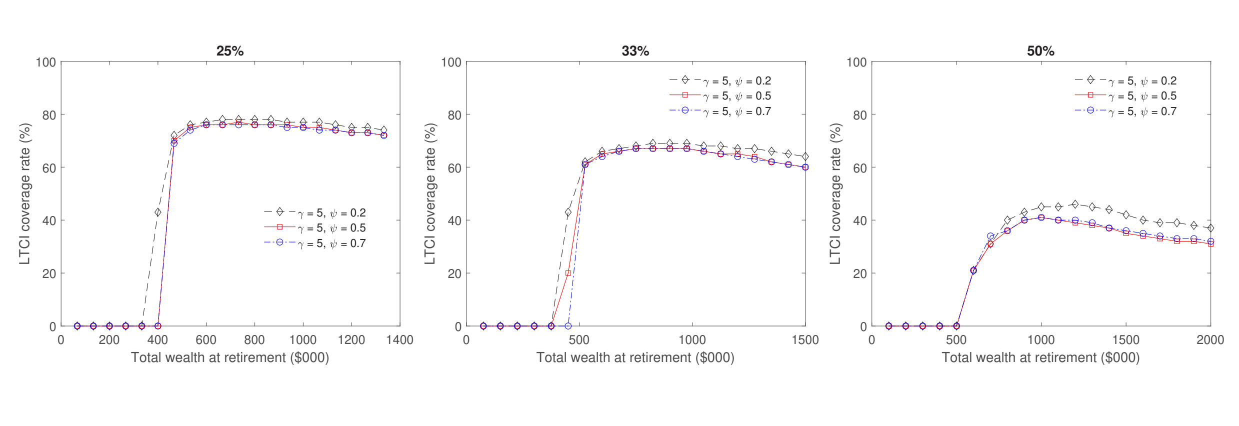

FIGURE E4. *Optimal LTCI coverage rates for retirees with different levels of the EIS and no bequest motives. The title above each panel denotes the proportion of total wealth in home equity at retirement. The life annuity is not offered in the market.*

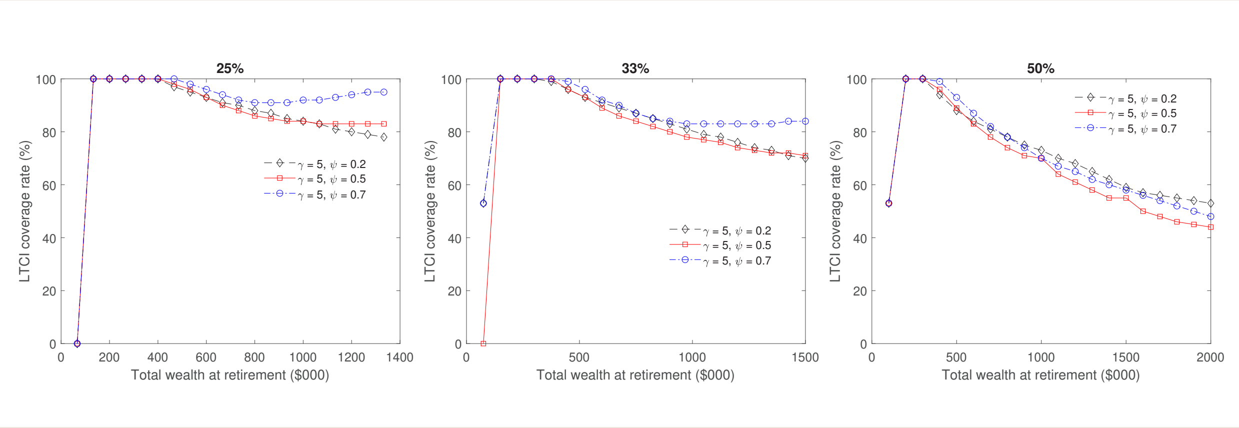

FIGURE E5. *Optimal LTCI coverage rates for retirees with different levels of the EIS and a certain bequest motive (b = 2). The title above each panel denotes the proportion of total wealth in home equity at retirement. The life annuity is not offered in the market.*

E.2. BOTH ANNUITIES AND LTCI

LTCI is known to enhance the demand for annuities in the absence of housing wealth (see, e.g., Ameriks et al. 2008; Wu, Bateman, and Stevens 2016). We find the same result in a wide range of wealth levels. Figure E.6 shows that retirees endowed with between $200,000 and $800,000 liquid wealth increase their optimal annuitization rates after having access to LTCI. Retirees in the higher wealth bands, however, optimally cut their annuitization rates to purchase LTCI.

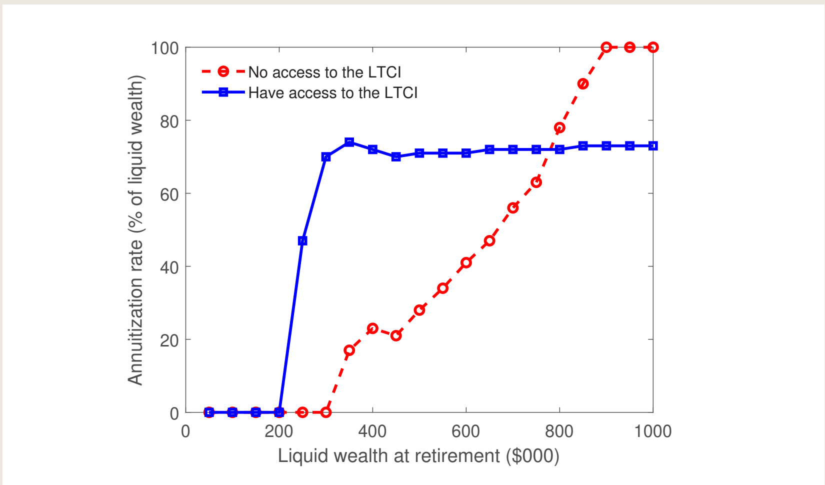

FIGURE E6. *Optimal annuitization rates with and without access to LTCI for retirees endowed with liquid wealth and no housing wealth at retirement. The preference parameters are b = 2, γ = 5, ψ = 0.5.*

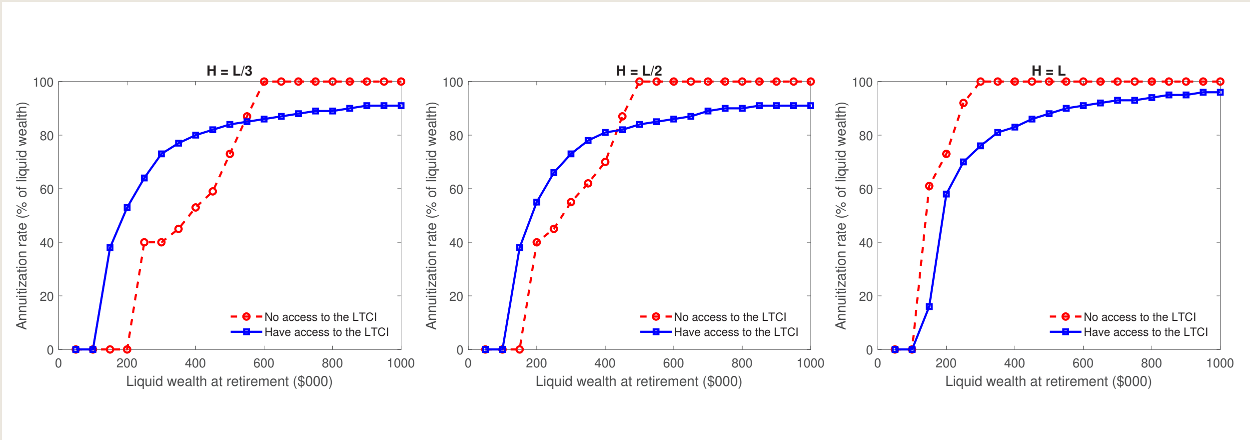

FIGURE E7. *Optimal annuitization rates for homeowners with and without access to LTCI. The title above each panel denotes the ratio of housing wealth (H) to liquid wealth (L) at retirement. The preference parameters are b = 2, γ = 5, ψ = 0.5.*

We also verify the result in Davidoff (2009) that the complementarity between life annuities and LTCI can be reversed by illiquid housing wealth. Figure E.7 compares the interaction between annuity demand and access to LTCI across different levels of housing wealth. As the ratio between housing wealth and liquid wealth increases, the improvement in the optimal annuitization rate brought about by LTCI shrinks before it disappears. This shows that housing wealth undoes the complementarity between the two products.

Figures E.8 and E.9 supplement the result shown in figure 28, and show that, for homeowners without bequest motives, the optimal annuitization rates vary little with risk aversion or the EIS.

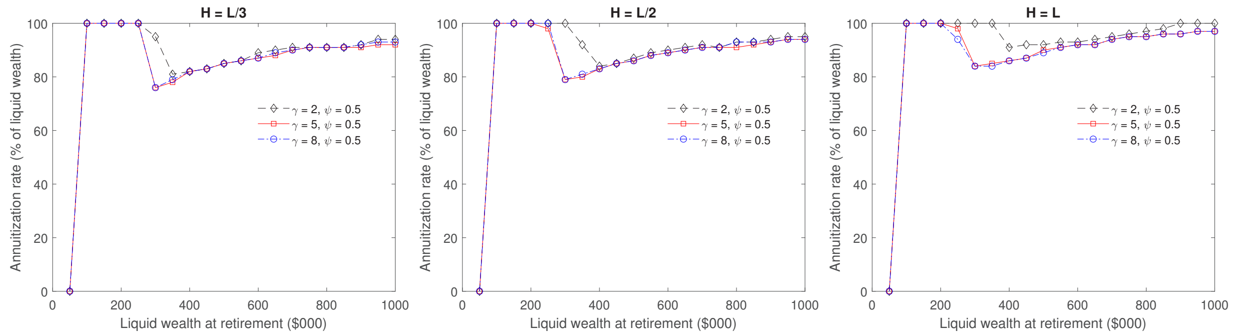

FIGURE E8. *Optimal annuitization rates for retirees with different levels of risk aversion and no bequest motives. The title above each panel denotes the ratio of housing wealth (H) to liquid wealth (L) at retirement. LTCI is offered in the market.*

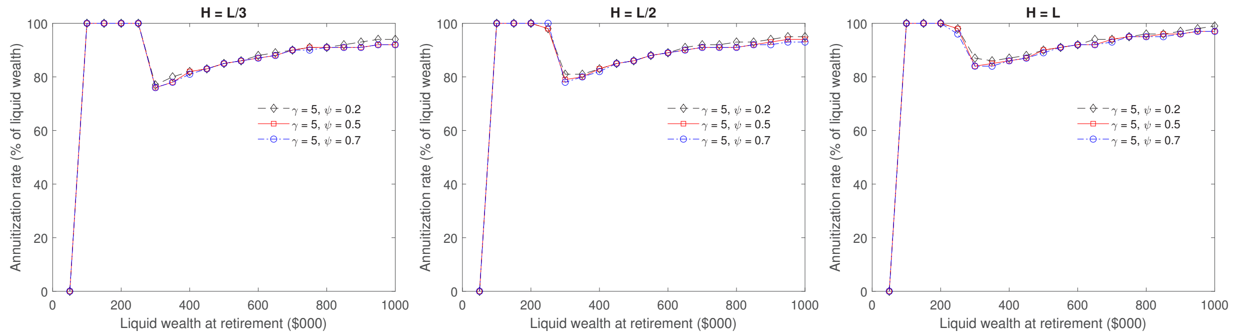

FIGURE E9. *Optimal annuitization rates for retirees with different levels of the EIS and no bequest motives. The title above each panel denotes the ratio of housing wealth (H) to liquid wealth (L) at retirement. LTCI is offered in the market.*

Figures E.10 and E.11 supplement the result shown in figure 29, and show that, for homeowners with bequest motives (b = 2), the optimal annuitization rates vary little with risk aversion or the EIS.

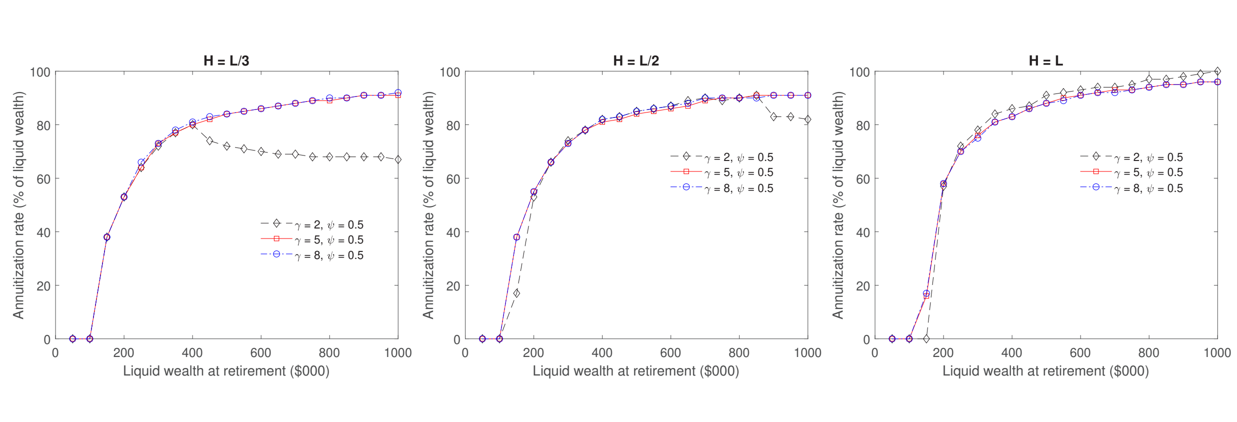

FIGURE E10. *Optimal annuitization rates for retirees with different levels of risk aversion and a certain bequest motive (b = 2). The title above each panel denotes the ratio of housing wealth (H) to liquid wealth (L) at retirement. LTCI is offered in the market.*

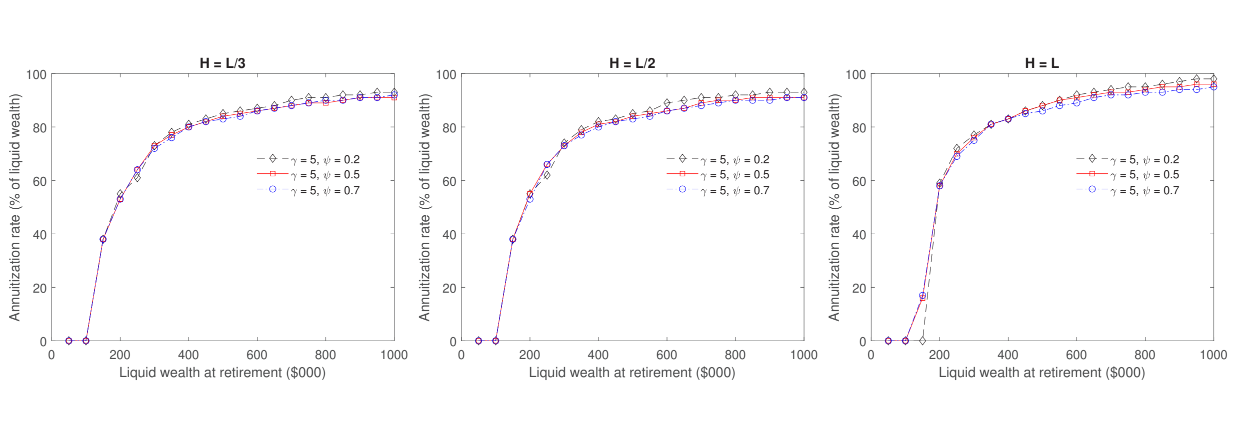

FIGURE E11. *Optimal annuitization rates for retirees with different levels of the EIS and a certain bequest motive (b = 2). The title above each panel denotes the ratio of housing wealth (H) to liquid wealth (L) at retirement. LTCI is offered in the market.*

Figures E.12 and E.13 supplement the result shown in figure 30, and show that, for homeowners without bequest motives, the optimal LTCI coverage rates vary little with risk aversion or the EIS.

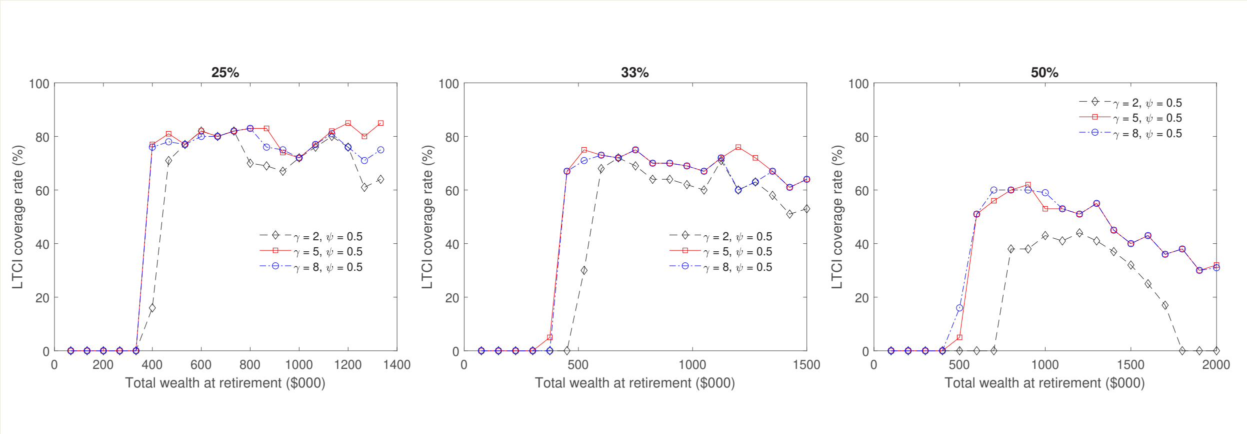

FIGURE E12. *Optimal LTCI coverage rates for retirees with different levels of risk aversion and no bequest motives. The title above each panel denotes the proportion of total wealth in home equity at retirement. The life annuity is offered in the market.*

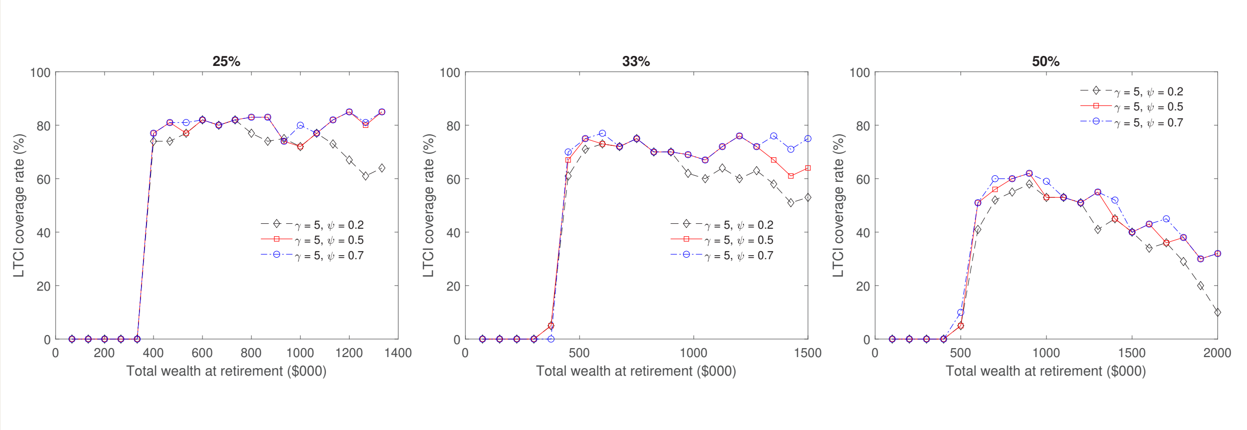

FIGURE E13. *Optimal LTCI coverage rates for retirees with different levels of the EIS and no bequest motives. The title above each panel denotes the proportion of total wealth in home equity at retirement. The life annuity is offered in the market.*

Figures E.14 and E.15 supplement the result shown in figure 31 and show that, for homeowners with bequest motives (b = 2), the optimal LTCI coverage rates vary little with risk aversion or the EIS.

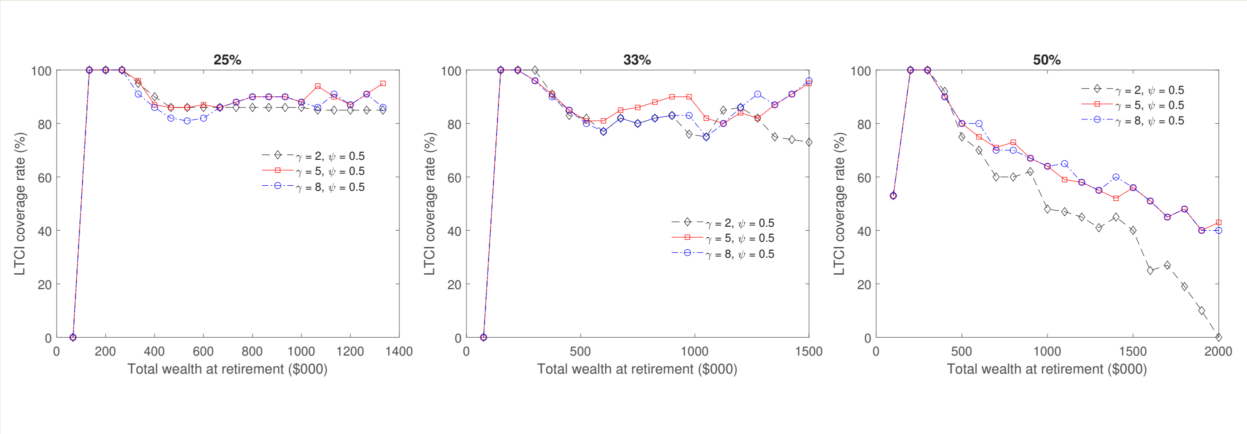

FIGURE E14. *Optimal LTCI coverage rates for retirees with different levels of risk aversion and a certain bequest motive (b = 2). The title above each panel denotes the proportion of total wealth in home equity at retirement. The life annuity is offered in the market.*

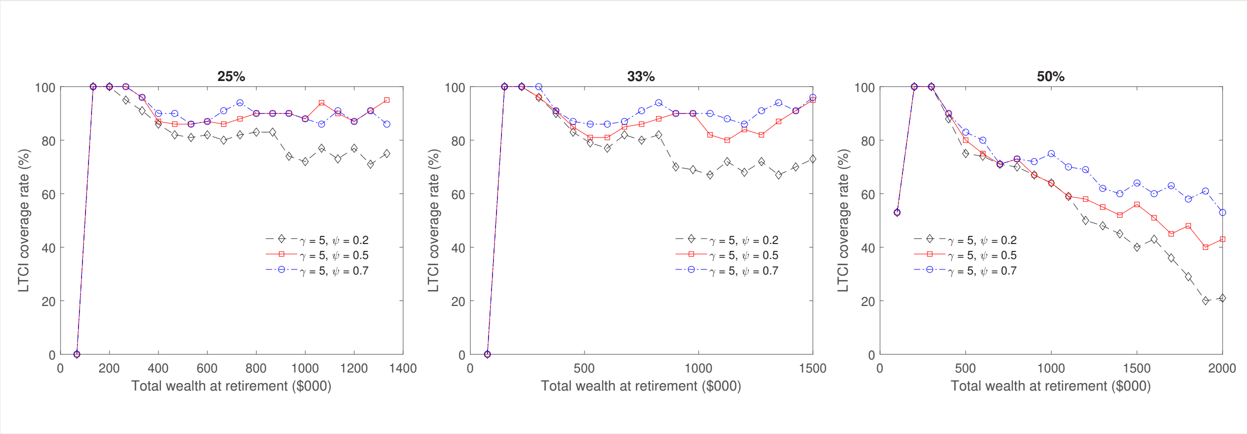

FIGURE E15. *Optimal LTCI coverage rates for retirees with different levels of the EIS and a certain bequest motive (b = 2). The title above each panel denotes the proportion of total wealth in home equity at retirement. The life annuity is offered in the market.*

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