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ABSTRACT

Recent market volatility and low bond yields have left many investors considering new ways to capture market upside while limiting their losses. One potential approach would be to use options (e.g., calls and puts), either through a direct purchase or through some type of prepackaged product, an approach we generalize with the term "protected wealth strategy" (PWS). PWSs effectively reshape the potential return distribution of an underlying financial instrument, such as the S&P 500, which some investors find attractive. Using a utility-based resampled optimization framework, we find that PWSs have the potential to improve portfolio efficiency—potentially significantly—depending on the strategy attributes and investor circumstances. This is especially true of strategies that involve selling out-of-the-money put options (i.e., buffer approaches). Before implementing any type of PWS, though, an investor needs to understand the unique risks and costs associated with each respective strategy, especially when considering a prepackaged product. A PWS can be implemented by a household (or by a financial advisor) through the direct purchase and sale of individual options. Alternatively, there are prepackaged versions available, such as a fixed indexed annuity (FIA) or registered index-linked annuity (RILA), which we collectively refer to as a prepackaged protected wealth strategy (P-PWS). An FIA provides principal protection (i.e., no downside risk) with some potential upside, while RILAs can provide more upside in exchange for the policyholders' willingness to absorb some risk of loss.

IT'S GOOD TO HAVE OPTIONS: THE POTENTIAL BENEFITS OF ALLOCATING TO PROTECTED WEALTH STRATEGIES

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INTRODUCTION

Recent market volatility and low bond yields have left many investors considering new ways to capture market upside while limiting their losses. One potential approach would be to use options (e.g., calls and puts), either through a direct purchase or through some type of prepackaged product, an approach we generalize with the term "protected wealth strategy" (PWS). PWSs effectively reshape the potential return distribution of an underlying financial instrument, such as the S&P 500, which some investors find attractive. Using a utility-based resampled optimization framework, we find that PWSs have the potential to improve portfolio efficiency—potentially significantly—depending on the strategy attributes and investor circumstances. This is especially true of strategies that involve selling out-of-the-money put options (i.e., buffer approaches). Before implementing any type of PWS, though, an investor needs to understand the unique risks and costs associated with each respective strategy, especially when considering a prepackaged product. A PWS can be implemented by a household (or by a financial advisor) through the direct purchase and sale of individual options. Alternatively, there are prepackaged versions available, such as a fixed indexed annuity (FIA) or registered index-linked annuity (RILA), which we collectively refer to as a prepackaged protected wealth strategy (P-PWS). An FIA provides principal protection (i.e., no downside risk) with some potential upside, while RILAs can provide more upside in exchange for the policyholders' willingness to absorb some risk of loss.

P-PWSs are often dismissed by households or advisors over concerns about commissions, lock-up periods, liquidity, and so on. However, it is important to contrast (and understand) the potential benefits of a strategy versus the efficacy of the products available to implement the approach. There are obviously several important considerations to review before purchasing any type of financial product; however, the P-PWSs product category continues evolving (e.g., there are an increasing number of fee-only products available in the space) and these products can be more attractive depending on household preferences (e.g., in households where liquidity is less of an issue).

The potential benefits of PWSs are determined using a resampled utility optimization framework, based on the Constant Relative Risk Aversion (CRRA) utility function. More-traditional optimization routines, such as a mean variance optimization, are not necessarily appropriate when determining optimal allocations to PWSs, given the non-normal return distributions associated with the strategies. While other metrics, such as semi-standard deviation, could be used to determine allocations, a utility approach is used because it more easily allows for the incorporation of varying risk-aversion levels.

The analysis considers four relatively plain-vanilla PWSs: a 0% floor strategy, a 10% floor strategy, a 10% buffer strategy, and a 20% buffer strategy, all with the same underlier: the S&P 500. The assumed participation rate is assumed to be 100% up to the cap,[1] which is based on the estimated options budget determined using the Black-Scholes pricing model. These four strategies are by no means exhaustive and are selected to reflect commonly considered strategies that provide some upside while limiting downside. More-common options strategies, such as selling covering calls, are not considered since the approaches typically do not seek to limit losses (i.e., provide downside protection).

The primary assumptions of the analysis reflect today's challenging investment environment, with relatively high implied volatility (25%) (Cboe.com n.d.) and relatively low interest rates (i.e., the assumed yield on the 1-year US Treasuries is 0.5%); however, a number of key assumptions are varied to determine how the optimal allocation to PWSs is impacted.

The results of the optimizations suggest that PWSs can improve portfolio efficiency, potentially significantly, although the benefits depend considerably on the strategy attributes and investor circumstances (e.g., risk-aversion level). Of interest to financial advisors, we find that PWSs are more attractive for risk-averse or moderately risk-averse investors. PWS approaches that involve selling out-of-the-money put options (i.e., buffer approaches) have been especially attractive, given historical options pricing dynamics.

I. PROTECTED WEALTH STRATEGIES

PWSs could generally be defined as any type of investment strategy that provides some downside protection with some upside, where the return of the product is tied to some type of market instrument. This typically would be a stock index (e.g., the S&P 500), but hypothetically could also include individual securities, foreign currencies, and so on. These products have historically been referred to as structured products; however, structured products are often designed so there is no potential for loss (e.g., similar to an FIA). This does not accurately describe the strategies considered in this research, which have the potential for loss, but where the losses are generally muted (e.g., more consistent with a RILA).

PWSs are generally going to be built using some combination of financial options (e.g., calls and puts). Financial options are not a new financial instrument and early versions date back thousands of years to ancient Greece. Puts and calls have been prominent investment vehicles since the Dutch tulip mania of 1636, and have been used in trading in the United States since 1872. Options have become increasing popular in the United States, especially in the retail space since the Chicago Board of Options Exchange (CBOE) and Options Clearing Corporation (OCC) formed in 1973.

P-PWSs have also been around for decades and vary materially by structure. For example, if the P-PWS is issued as a note, the principal and market return are subject to the issuer's creditworthiness for payment of all amounts (as a senior, unsecured debt obligation of the issuer). This fact is especially noteworthy for investors who held an estimated $18.6 billion in face value P-PWSs issued by Lehman Brothers in September 2008. While many of these P-PWSs were marketed as "100% Principal Protected…Notes," the investors in the Lehman Brothers stock prices may eventually end up receiving only 21 cents on the dollar for each dollar originally invested in the products. A second type of SP structure is market-linked CDs, which are FDIC-insured variable rate certificates of deposit, and that therefore have FDIC principal protection up to certain limits.

Perhaps the most well-known P-PWSs exist in the annuity space, such as an FIA, which were first introduced in the 1990s. An FIA provides principal protection (i.e.,

1. For readers not familiar with the term "cap," it is the maximum potential return you can earn over the period. For example, if the cap is 10%, and the return of the underlier (e.g., the S&P 500) over the period is 15%, the return is capped at 10%.

no downside risk) with some potential upside. RILAs, which have also been called a structured annuity, buffered annuity, or hybrid annuity, are relatively new and provide more potential upside than FIAs in exchange for the policyholders' willingness to absorb some risk of loss. While FIAs have been relatively popular historically, low interest rates create a challenging environment for products that attempt to offer principal protection and the possibility of upside, an effect documented in section II. Also, while most annuity products are sold with commissions (which are often associated with surrender penalties) there is a rising number of products and firms offering fee-only versions.

There are several things an individual should consider before purchasing a P-PWS, such as credit risk, lack of liquidity, a high level of complexity, unique tax considerations, and so on. For example, P-PWSs often require a capital commitment for some minimum number of years (e.g., a 2- to 10-year term), where access to principal and gains can be limited. Early access or withdrawals can trigger significant penalties (e.g., surrender penalties); however, P-PWSs typically have higher options budgets (e.g., more upside) than PWSs that could be constructed by a household (e.g., assuming government bonds are the risk-free rate).

The tax treatment of PWSs (and P-PWSs) can be relatively complex in a taxable account. Some products have imputed interest and gains are typically taxed at ordinary income rates, which can potentially make them relatively less attractive than equities on an aftertax basis (e.g., where dividends and gains can be taxed at long-term capital gains rates). While taxes are obviously an important concern for investors (seeking to implement them in a taxable account), given the underlying differences in taxation approaches and general complexity, taxes are ignored for this analysis.

II. DECOMPOSING PROTECTED WEALTH STRATEGIES

PWSs can generally be decomposed into some combination of call and put options that vary depending on the strategy. The assembly process for a PWS with a 0% floor is displayed in exhibit 1.

The interest rate from the bond (i.e., the zero coupon) helps set the options budget (i.e., the upside). For example, if the zero coupon (which generates the guaranteed return of principal) has a yield of 3%, we assume the investor can use approximately 3% of the principal to purchase options to gain upside exposure (i.e., the options budget). The upside is determined by the cost of difference of an at-the-money call option and out-of-the-money call options. An at-the-money call option would be ***purchased*** and an out-of-the-money call option would be ***sold*** so that the total cost of the options would be 3%. The upside exposure would be defined based on the strike price of the out-of-the-money option.

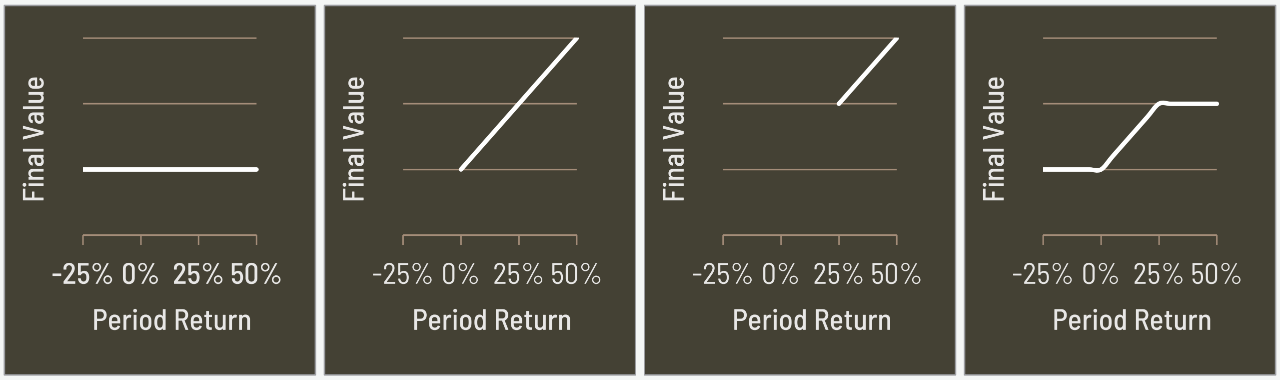
Options prices can be obtained in real time on various exchanges and a significant amount of historical data are available online. For this research we primarily rely on the Black-Scholes options pricing model (see Black and Scholes 1973; Merton 1973), which uses a partial differential equation that can be used to describe the price of a European option over time. We expand on this model, as well as on some of the key assumptions used for the analysis, in appendix 1.

One of the most important variables in the Black-Scholes formula is implied volatility. Implied volatility is not directly observable and instead is implied by the market price of the option (hence the name). In other words, implied volatility is the volatility (i.e., input) that yields a theoretical value for the option equal to the current market price of that option when using in a given pricing model (such as Black-Scholes). Note that, although implied volatility has been historically correlated to past (realized) volatility, it is intended to be a forward-looking measure.

One of the most widely followed implied volatility measures is the VIX index, which is also commonly referred to as the fear index. Originally tied to the S&P 100, the metric changed to tracking the S&P 500 in 2003. The VIX is an implied volatility index created by the CBOE that measures the market's expectation of 30-day S&P 500 volatility implicit in the prices of near-term S&P 500 options. VIX is quoted in percentage points, just like the standard deviation of a rate of return. CBOE has S&P implied volatility indexes with terms ranging from 9 days (the Cboe Short-Term Volatility Index) to 1 year (Cboe S&P 500 1-Year Volatility Index).

Exhibit 2 includes historical values for the VIX index, the Cboe S&P 500 1-Year Volatility Index (which has a January 2007 inception date), and the historical implied volatility of 1 year at-the-money S&P 500 Call Options

EXHIBIT 1: *Assembling a Protected Wealth Strategy with a 0% Floor*

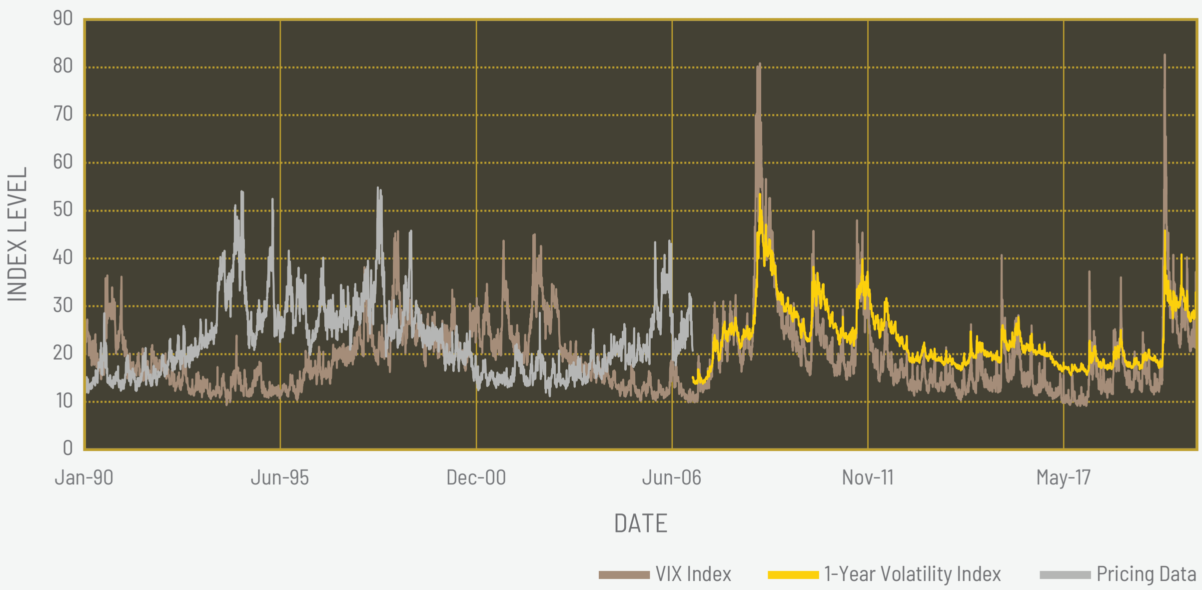


based on data obtained from DeltaNeutral.com, to provide some perspective as to how the indexes have varied over time.

Implied volatilities tend to increase in response to market volatility. Implied volatilities calculated from identical call and put options have often been empirically found to differ, although they should be equal in theory (see Ahoniemi and Lanne 2009). There is an inherent demand for put options that does not exist for similar calls, since institutional investors buy puts regularly for purposes of portfolio insurance. There is often no market for investors looking to sell the same options to offset this demand, meaning that prices may need to be bid up high enough for market makers to be willing to become counterparties to the deals. For the purposes of this analysis, though, the implied volatilities for puts and calls are assumed to be identical.

Realized volatility has typically been lower than implied volatility, as demonstrated in exhibit 3, which includes historical VIX levels along with historical realized 30-day

EXHIBIT 2: *Historical Implied Volatility, 1990–2020*



volatility. Realized volatility is estimated using the same approach by S&P for their realized volatility indexes (S&P Dow Jones Indices 2021) and is technically based on the 21-day absolute rolling deviation in S&P 500 price returns.

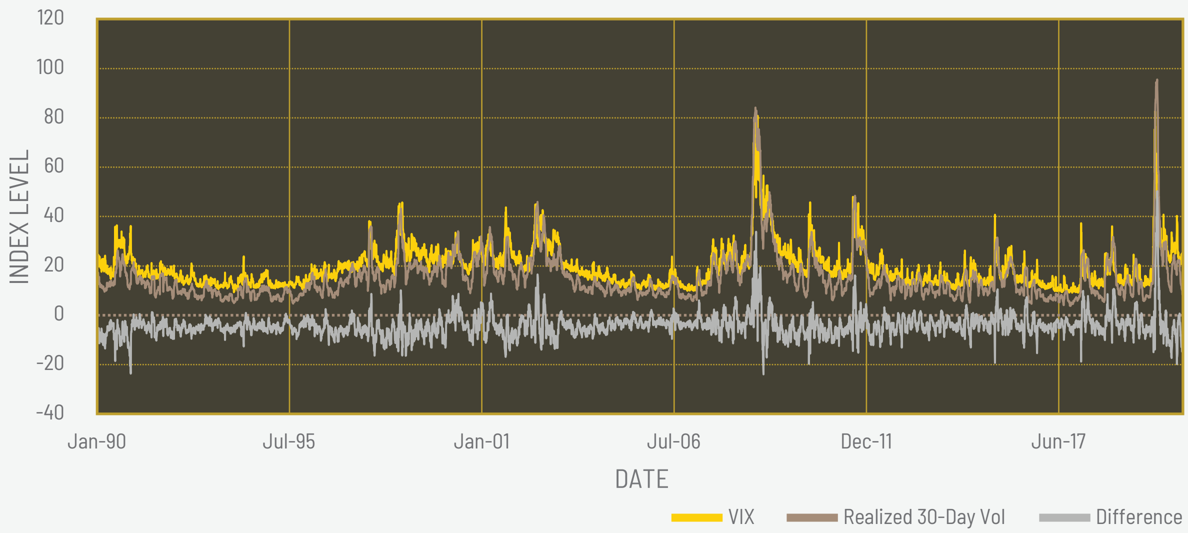
The fact that implied volatility has been higher-than-realized volatility suggests that there is an implicit cost for buying options. It is also worth noting that implied volatilities are not constant across strike prices, and that they tend to increase for out-of-the-money put options and in-the-money call options, as demonstrated in exhibit A1.1 (in appendix 1).

These differences are important since they provide context around the potential benefits of buying or selling options (i.e., risk). Since options are priced based on higher-than-realized volatility levels, there is a potential benefit to selling options versus buying them, especially out-of-the-money put options. This has important implications for which types of PWSs may be the most attractive and is a topic we explore in greater depth in section III.

Higher recent implied volatility levels have been accompanied by a notable decline in bond yields. We demonstrate this effect for 1-year government bonds (Federal Reserve Bank of St. Louis 2021a), the 1-year LIBOR rate (FRED 2021b), and the yield on Moody's Seasoned Aaa Corporate Bonds in exhibit 4 (FRED 2021c). The 1-year LIBOR rates are included to demonstrate their similarity to 1-year government bond yields, since LIBOR rates are a more common assumption in options pricing models (although we use the government yields for our analysis as a simplifying assumption). Aaa yields are included since they are a better proxy for the additional yield available through the issuers of P-PWSs (e.g., insurance companies), which can result in a higher options budget. Lower bond yields, coupled with higher recent implied volatility, has created a challenging environment for investors who want a PWS that offers upside with no downside (i.e., a 0% floor), which would be the return profile of an FIA, for example. With 1-year government yields close to 0%, there is effectively no budget to purchase options for households that want upside with principal protection; however, using other estimates of the options budgets (i.e., Aaa yields, which reflect the potential options budget of a P-PWS) there is still some available upside.

We demonstrate how cap rates have evolved for 1-year terms from December 1999 to December 2020 on the S&P

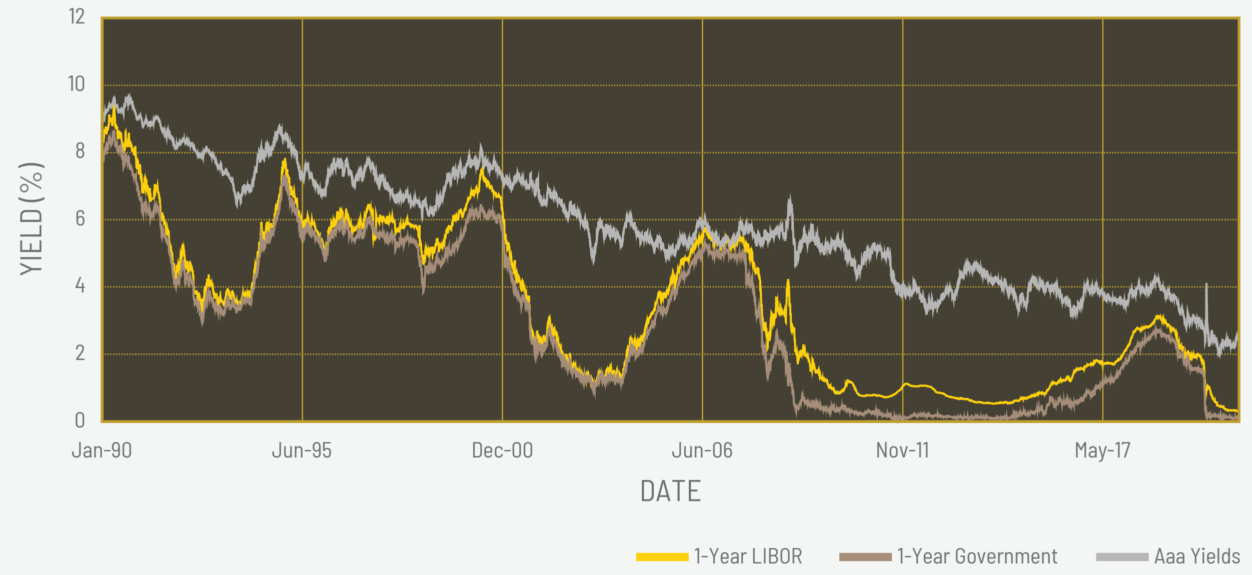
EXHIBIT 3: *Historical Implied and Realized 30-Day Volatility, 1990–2020*



500 in exhibit 5, where the options budget (to determine the cap rate) is based on either 1-year government bonds or Moody's Seasoned Aaa Corporate bond yields. Options prices are based on actual historical data obtained from DeltaNeutral.com on the S&P 500 Index. Since options are not generally available that have a precise 1-year expiration, we use interpolation and run a series of regressions to estimate the rolling to cost of 1-year options. The maximum assumed cap is 30%.

The floor, which is the maximum potential loss over the period, is assumed to be 0% (which is consistent with FIAs). The assumed participation rate is 100% up to whatever cap is available based on the options budget.[2] Note that the results are somewhat volatile (with lots of spikes), given some of the noise associated with the options pricing data. Therefore, the reader should focus more on the general trend versus single point-in-time estimates.

EXHIBIT 4: *Historical Bond Yields, 1990–2020*



The 1-year government yields reflect the risk-free rate available to retail investors (i.e., households). Aaa yields are included to reflect the higher options budget for organizations that create P-PWSs, such as insurance companies, which can generally invest for a longer potential term and use corporate bonds.

As bond yields have declined over the past two decades, the potential caps for a 0% floor PWS have decreased significantly, especially when we assume that the options budget is based on 1-year government bonds. For example, as of December 31, 2020, the caps available stood at 0.2% and 4.0%, based on 1-year government bonds and the Moody's Aaa yields, respectively.

By accepting some downside risk, though, an investor can capture more potential upside. For example, if investors are willing to lose up to 10%, they could implement a 10% floor strategy by selling an at-the-money put option and buying an out-of-the-money put option with a strike of 90% of the underlier. Alternatively, they could implement a 20% buffer strategy by selling a put option with a strike of 80% of the underlier.[3] Both of these strategies subject the investor to some downside risk, but will generate a premium that can be used to potentially purchase call options and create additional upside. Exhibit 6 pro-

2. The participation rate is the percentage of the underlier return that will be credited for the strategy. For example, if the participation rate is 50% (100%) and the market goes up 20%, you would earn 10% (20%).

vides some context as to what the historical premiums would be for these two approaches.

The average premiums from the 10% floor and 20% buffer strategies have varied over time, but have averaged around 3% of the principal. Exhibit 7 provides context as to what kind of upside could be created using this premium coupled with the yield of the respective bond for the 10% floor approach.

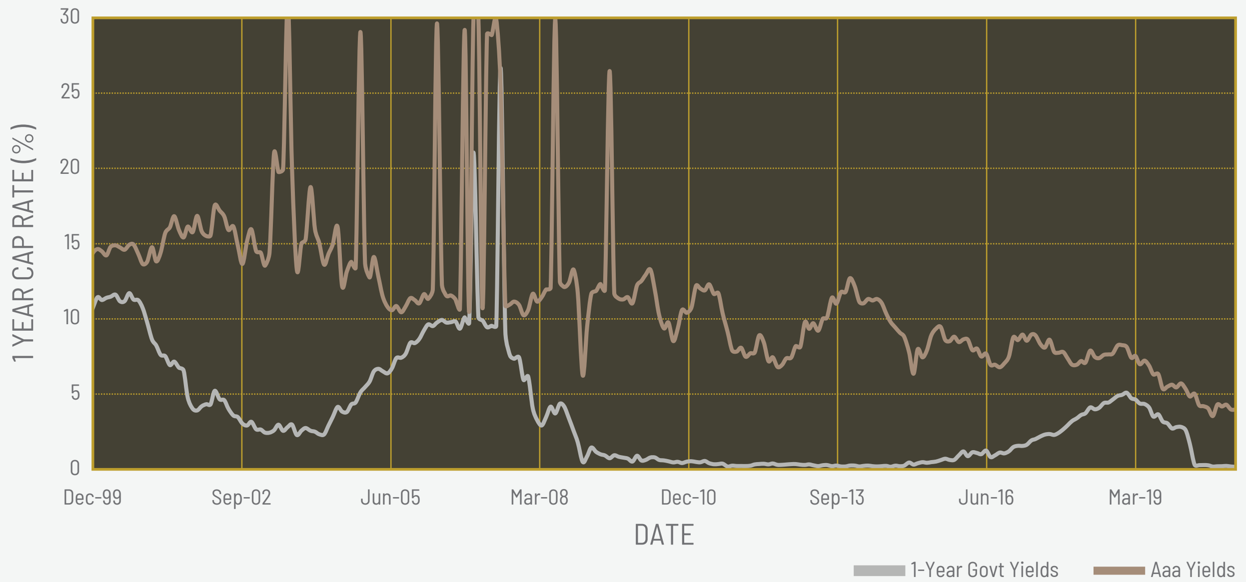
With a 10% floor, the cap rates based on 1-year government bonds and the Moody's Aaa stood at 6.1% and 11.0%, respectively as of December 31, 2020. It is not clear, though, whether an investor should consider these PWSs, and if so, which type of investor should consider them. This is the question we seek to answer in our subsequent analysis.

III. OPTIMAL PROTECTED WEALTH STRATEGY ALLOCATIONS

When thinking about the potential role of a PWS within a total portfolio framework, it is important to note that a PWS is not necessarily a new asset class but is typically going to be a derivative of an existing asset class.[4] For example, if the underlier is the S&P 500, the returns of the PWS would depend on the (price) return of the S&P 500 along with whatever combinations of options represent the PWS exposure. For the PWS to improve the efficiency of a portfolio, it would need to somehow reshape the return distribution in a way that makes it relatively attractive to the other available options.

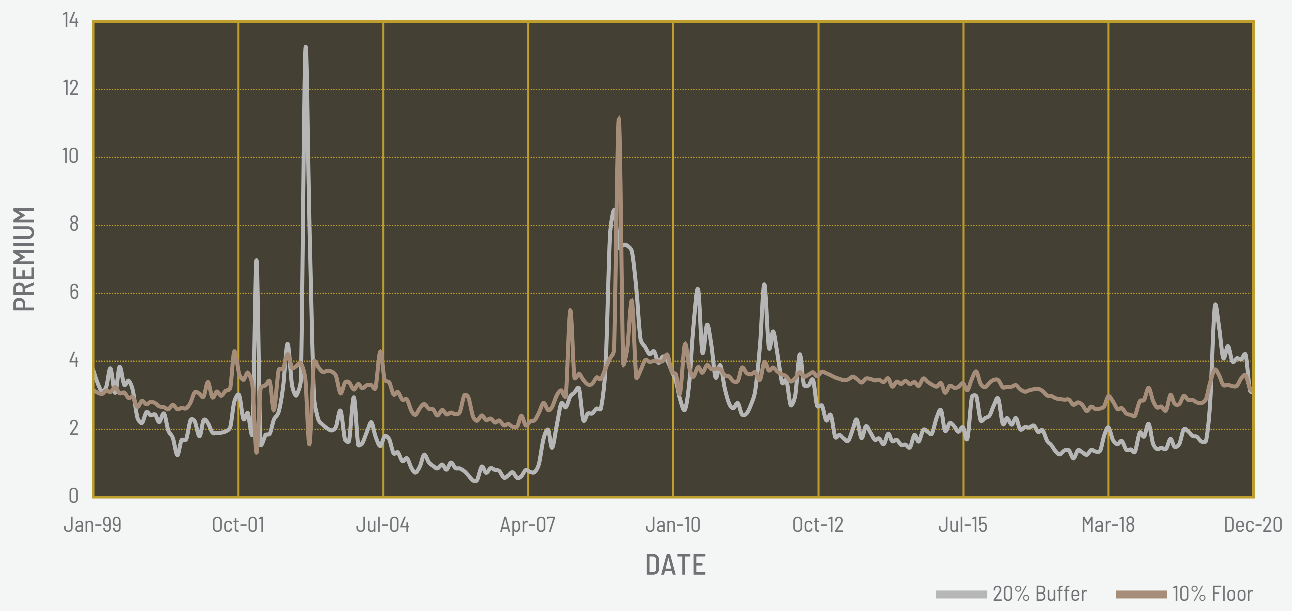
While variance (and standard deviation) is perhaps the most widely used definition of risk when it comes to building portfolios, largely due to the influential work of Markowitz (1952) among others, PWS returns are not typically going to follow a normal distribution. This makes variance (and traditional Mean Variance Optimization) a relatively poor method to determine the potential benefit of a PWS. Therefore, we use a utility-based approach based on the Constant Relative Risk Aversion (CRRA) utility function, as noted in equation 1, since it considers the entire distribution of potential returns, along with the investor's risk-aversion level:

EXHIBIT 5: *Historical Cap Rates on 1-Year S&P Protected Wealth Strategies, 0% Floor, 2000–20*



3. A buffer is the first amount of losses absorbed by the strategy. For example, if the buffer is 20% (10%), and the return of the underlier over the period is –30%, you would lose only 10% (20%).

EXHIBIT 6: *Historical Premium on 1-Year S&P Protected Wealth Strategies, 20% Buffer, and 10% Floor, 2000–20*



EQUATION 1:

$$u(x)= x^{1-\gamma}$$

For those readers not familiar with utility functions, they are used to quantify outcomes and preferences. A key component of utility, in particular the CRRA utility function, is the concept of diminishing marginal utility, which means the first unit of consumption (x) of a good or service yields more utility than the second and subsequent units.

Utility functions are ideal for analyzing a non-normal return distribution, since each value can be considered and weighted based on risk aversion. The level of risk aversion (γ) describes the penalty associated with a bad outcome, where higher levels of risk-aversion levels would increasingly penalize bad incomes (i.e., negative returns).

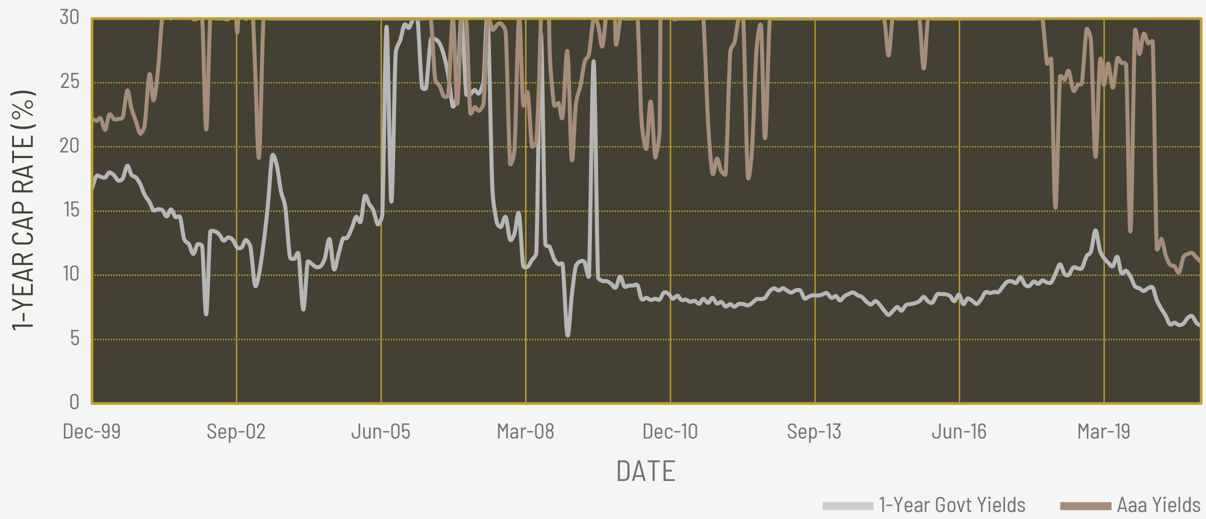
For this analysis we vary the risk-aversion factor (γ) from 0 to 20 in increments of 1.[5] A moderately risk-averse investor is typically assumed to have a risk-aversion coefficient (γ) of approximately 4. We explore how portfolio allocations vary by different risk-aversion levels in section IV; for reference purposes, however, the optimal equity allocation associated with risk-aversion levels of 1, 2, 4, 8, and 20 were approximately 90%, 70%, 50%, 30%, and 10%, respectively.

The goal within each optimization is to maximize the certainty equivalent utility for some potential weights to the respective opportunity set. The actual optimal allocations are determined using a series of resampled optimizations. We perform resampling to reduce the potential impact of estimation error. We perform 20 separate optimizations, each consisting of 50 years of returns. The optimal allocations are defined as the average weights to the opportunity set across the 20 separate optimizations. The seed values for each of the 20 optimizations are constant to ensure the same data are used across simulations.

A maximum of seven investments are potentially includ-

4. Depending on the underlier.

EXHIBIT 7: *Historical Cap Rates on 1-Year S&P Protected Wealth Strategies, 10% Floor, 2000–20*



ed in the opportunity set for the optimizations: US cash (i.e., a risk-free asset), US bonds, non-US bonds, US large cap equity, US small cap equity, non-US equity, and the respective options-based strategy being considered (i.e., the PWS).

The base capital market assumptions (CMAs) for the respective asset classes are included in exhibit 8. These are similar to Morningstar's 2021 CMAs and reflect the market environment as of December 31, 2020, which is consistent with the other key assumption date for the analysis.

We assume returns follow a multivariate normal distribution when creating 50-year return series for the optimizations. The analysis is a one-period model, which is assumed to be 1 year.

The equity proxy for the PWS is US large cap equity, which is effectively the S&P 500. The assumed dividend yield for equities is 2%, which is slightly above the dividend yield on the S&P 500 as of December 31, 2020 (which was 1.5%) but is similar to the average dividend yield of the S&P 500 from 2000 to 2020 (which was 1.9%).

Worth noting: a 2% assumed dividend yield is well below the historical long-term US average, which was approximately 4.3% from 1871 to 2020 based on the Shiller dataset. The lower overall dividend yield reflects the increase in share repurchases in the United States, an increase that began in the 1970s and had become the predominant way companies return money to shareholders by the mid-2000s (Straehl and Ibbotson 2017). Dividend yields are an important assumption for the Black-Scholes calculations, as well (detailed in appendix 1).

The base implied volatility assumption for the Black-Scholes model is 25%, which is slightly below the level of the Cboe S&P 500 One-Year Volatility Index as of December 31, 2020 (27.6%), but higher than the longer-term average (Cboe.com n.d.). The assumed volatility (i.e., standard deviation) for the stock market index (i.e., the S&P 500) for the analysis is assumed to be 18% (see exhibit 8), which is below the implied volatility but more consistent with the long-term average. Note that implied volatility has historically been between 2% and 5% higher than realized volatility, depending on the historical period and respective proxy (e.g., the Cboe S&P 500 One-Year Volatility Index or the observed values from the delta neutral dataset). In addition to a 25% implied volatility level we also consider a 20% implied volatility level for robustness purposes.

The return on cash (i.e., the risk-free asset) is assumed

5. Technically, if the risk aversion level is 1 it is assumed to be 1.01 so the exponent is not 0.

EXHIBIT 8: *Base Asset Class Capital Market Assumptions*

#	ASSET CLASS	TOTAL RETURN	STANDARD DEVIATION	#	CORRELATIONS					
					ASSET CLASS #					
					1	2	3	4	5	6
1	US Cash	0.5	0.0	1	1.00	0.00	0.00	0.00	0.00	0.00
2	US Bonds	1.5	6.0	2	0.00	1.00	1.53	0.17	0.05	0.11
3	Non-US Bonds	1.5	9.0	3	0.00	0.53	1.00	0.15	0.10	0.49
4	US Large Cap Equity	6.0	18.0	4	0.00	0.17	0.15	1.00	0.79	0.65
5	US Small Cap Equity	8.0	24.0	5	0.00	0.05	0.10	0.79	1.00	0.62
6	Non-US Equity	6.5	20.0	6	0.00	0.11	0.49	0.65	0.62	1.00

to be the yield on 1-year government bonds and is assumed to be guaranteed (i.e., why it has a 0% assumed standard deviation in exhibit 8). This is the same interest rate used for the Black-Scholes pricing model (i.e., the implicit assumption is that the yield is the same as the 1-year LIBOR, which is consistent with the general historical relation noted in exhibit 3).

Four different types of PWSs are considered for the analysis. The first two PWSs are based on a floor strategy, either at 0% or 10%, and the second two PWSs are based on a buffer strategy, either 10% or 20%. The underlying option prices for each strategy are determined using the Black-Scholes pricing model. The caps for the PWS strategies using the base assumptions for the analysis are approximately 1%, 10%, 20%, and 10%, respectively, for informational purposes.

The approaches considered in this paper are relatively simple and could easily be recreated by a sophisticated investor or financial advisor with over-the-counter options. The strategies are also roughly consistent with the common approaches in P-PWSs today, such as FIAs (0% floor), RILAs (10% floor, 10% buffer, and 20% buffer), or other products (e.g., buffered exchange traded funds [ETFs]). We do not consider a covered call strategy, despite its popularity (and the historical literature on the approach), because it provides no explicit downside protection.

Exhibit 9 demonstrates the potential returns of the various PWSs against the price return of the underlier (again, which is assumed to be the S&P 500 index). The total return index exceeds the returns of the options strategy when the returns are positive (and low) because the total return index includes dividend yields.

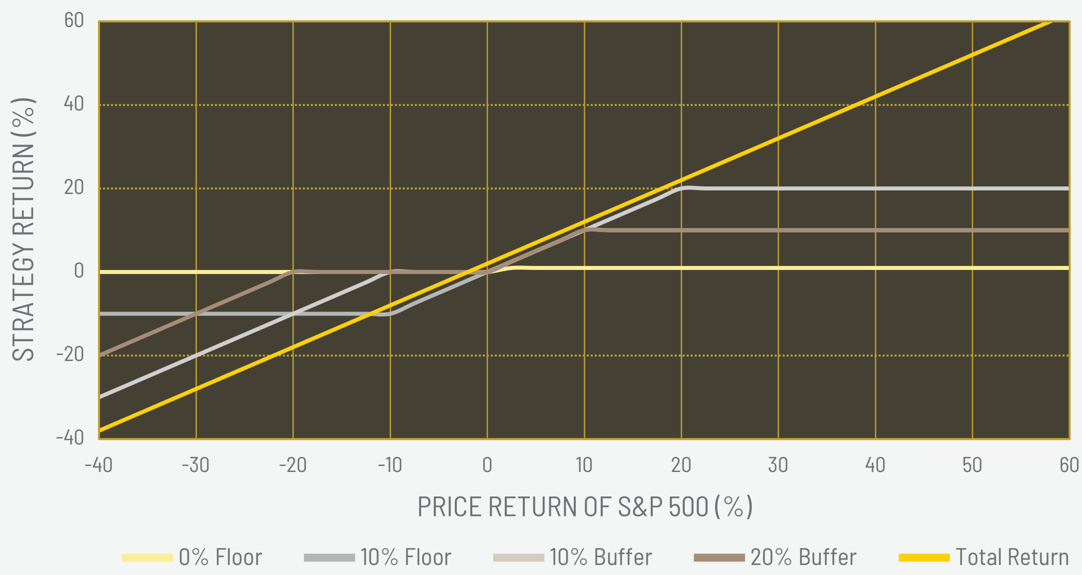
There is no assumed fee (or cost) to implement any of the strategies. This assumption is consistent given the wide availability of trading securities, including options, for little or no cost on online brokerages today. Additionally, it is possible to gain exposure to the market (e.g., stock and bond indices) using ETFs at an incredibly low cost.

The base analysis effectively assumes the PWSs are implemented by a household, since the options budget is based on the risk-free rate (cash), which is 1-year government bonds. P-PWSs can potentially offer higher potential payouts/caps than similar strategies used by households, if a larger options budget is assumed (see exhibits 4 and 5), since the strategies typically exist for a number of years (e.g., 5 years) where access to the funds is limited. Even P-PWSs that do not have a commission (e.g., those sold by fee-only advisors) typically have some type of surrender penalty if the P-PWS is liquidated before the term, along with a potential market value adjustment. Therefore, the additional potential benefits potential available through P-PWSs (i.e., higher cap rates) are not generally without some costs (which may be implicit) and need to be weighed before any type of purchase.

While PWSs can potentially offer more-attractive return distributions than traditional long-only investments, the diversification benefits of PWSs are going to vary significantly depending on the underlier. For example, the underlier for the PWSs in this analysis is based on the S&P 500. This means the correlation between the PWSs and the return of the S&P 500 is going to be relatively high. For example, the correlation between the 10% floor and the S&P price return is approximately .85 when the underlier is negative and .70 when the underlier is positive, whereas the correlation between the 20% buffer and the S&P price return is approximately .60 when the underlier is negative and .85 when the underlier is positive. However, these correlations vary significantly depending on the PWS assumptions (e.g., the options budget). Therefore, the positive potential attributes of the reshaped return distribution are going to be at least partially offset by the higher relative correlation to risky assets compared to other safe assets available (e.g., fixed income).

One obvious approach to improve the diversification benefits of the PWS would be to select an underlier with a lower market correlation, especially if that investment is not under consideration to be used in a more traditional portfolio. This will be a topic of future research.

EXHIBIT 9: *Strategy Returns vs. Price Return of the S&P 500*



IV. OPTIMIZATION RESULTS

Exhibit 10 includes the optimal allocations to the asset classes where the PWSs are excluded from the analysis. This exhibit provides perspective on how the risk-aversion levels relate to optimal equity allocations.

The equity allocations range from approximately 10%, at a risk-aversion level of 20, to 100%, at a risk-aversion level of 0. In other words, the risk-aversion levels that are considered result in a spectrum of relatively risk-tolerant investors to relatively risk-averse investors. Covering the entire space of potential equity allocations is important since it is not necessarily clear for which types of investors PWSs fit best, especially since they generally have to assume some downside risk across the strategies considered.

The individual asset class weights might seem a little unintuitive and would likely be adjusted before being implemented by a client. For example, the equity weights are highest to US small equity, followed by non-US equity, and then US large equity. This is typically the reverse of allocation weights in client portfolios and is a reflection of the CMAs used for the analysis.

In exhibit 11 we provide allocation information for each of the four PWSs, where we aggregate the three fixed income and three equity allocations so that it is easier to differentiate the overall allocations.

The PWS allocation varied significantly by product structure, although the buffer strategies (panels C and D) clearly received higher allocations than the floor approaches (panels A and B). The 20% buffer strategy received the highest allocations, exceeding 60% for moderate risk-aversion levels (e.g., approximate risk-aversion levels of 6, which would correspond to an equity allocation of 35% if the PWSs are excluded), although the allocations were still significant even for relatively risk-averse investors who would typically invest almost entirely in cash. The 10% floor strategy received the lowest allocations, the highest being 2% for risk-aversion levels of 3, which correspond to an equity allocation of approximately 60% if the PWSs are excluded.

The buffer strategies can be said to have soaked up some of the equity allocation for the moderately risk-averse

EXHIBIT 10: *Optimal Allocations by Risk-Aversion Level, Excluding Protected Wealth Strategies*

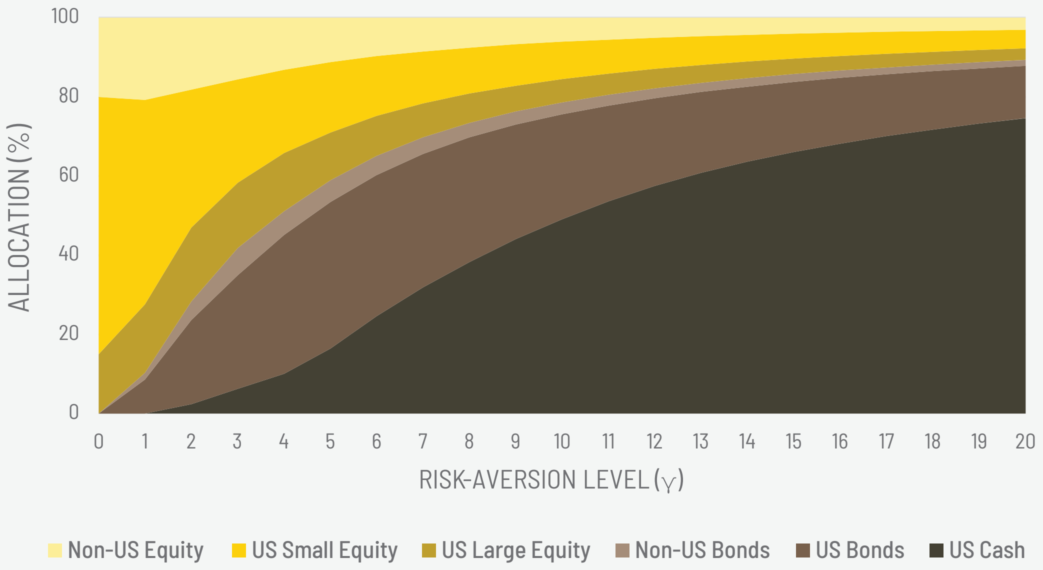
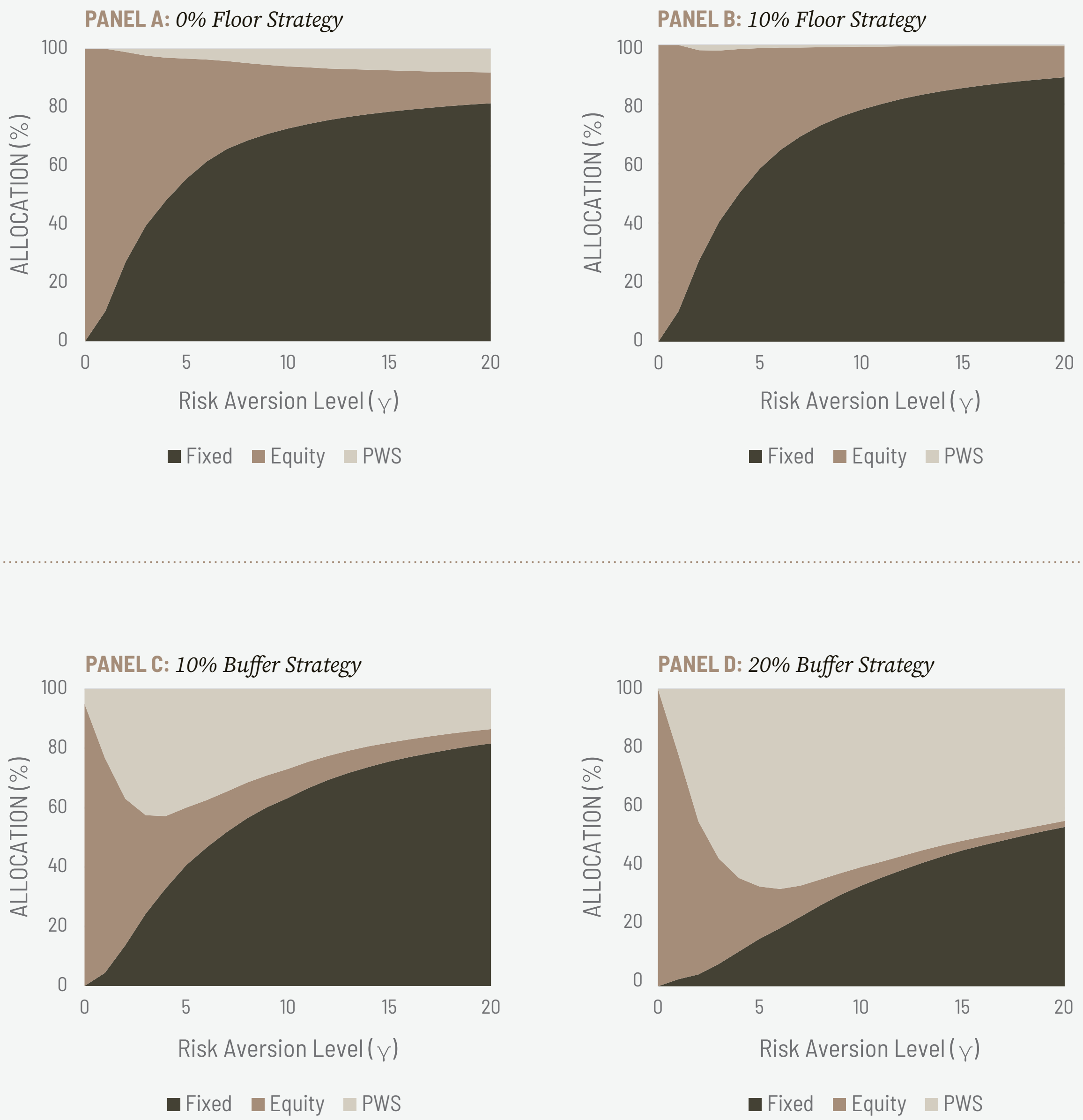


EXHIBIT 11: *Optimal Allocations by Risk Aversion Level for Protected Wealth Strategies*



investors. The buffer strategies are still risky, but they are less risky than directly owning the underlier (the S&P 500) given the buffer. A key reason the buffer strategies received significantly higher allocations than the floor strategies can be attributed to the cost structures implied within the options pricing model, which we explore more fully in section V.

Next, for robustness purposes we vary some of the key assumptions to determine what, if any, effect it would have on the results. We consider four key variables: the price return of equities (–4%, no change, and +4%), the assumed return on cash (no change, +1%, and +2%; note that this affects the investor's return, not the options budget), the options budget (no change, +1%, and +2%), and implied volatility (no change and –5%, which would be 20%).

Equity returns are varied to reflect the uncertainty surrounding the realized equity return, especially given current market valuations. The options budget is varied to reflect that PWS issuers can typically use rates that exceed government bonds to generate the options budget; however, there may also be additional fees that reduce the realized potential increase (and liquidity restrictions). The higher returns on cash are included to replicate the higher potential return available through various financial products, such as fixed rate annuities (also referred to as multiyear guaranteed annuities, or MYGAs) that have similar illiquidity features as certain P-PWSs (e.g., FIAs and RILAs), that can require dedicating money to a given strategy for some fixed period, such as 5 years. Implied volatility is adjusted to determine its potential impact on options pricing.

The allocations to the respective products are included in exhibit 12 using the base assumed implied volatility level (25%) for four risk-aversion levels: 2, 4, 8, and 20, which roughly correspond to equity allocation targets of approximately 70%, 50%, 30%, and 10%. The allocations for a 20% implied volatility level are included in appendix 2.

The PWS allocations increase as the options budget increases and with lower assumed returns on cash. The impact of the changes in the price return of equities was ambiguous based on the changes in other assumptions and the risk-aversion level, but is generally higher with higher equity returns.

The most notable change in allocations in exhibit 12 was for the 0% floor strategy. The allocations increased considerably for the 0% floor strategy when the options budget was increased, even by 1%, holding the other assumptions constant. If the increase in options budget for the 0% floor strategy is accompanied by a parallel increase in the expected return for the risk-free asset (e.g., assuming an alternative to the PWS is a MYGA) there is virtually no change in the allocation. In other words, the attractiveness of a 0% floor strategy (e.g., an FIA) is going to depend significantly on the assumed spread between the options budget and the return on cash.

The 10% floor strategy allocations did not generally change, even when the options budget was increased. This suggests the 10% floor strategy is dominated by some combination of cash and the other risk assets available, at least given the key assumptions of this analysis.

The results when the implied volatility is reduced to 20% are relatively similar compared to the base assumption (versus when implied volatility is 25% as in exhibit 9), which is why they are included in appendix 2.

Overall, the analysis suggests that PWSs can be attractive for investors, but the optimal allocations can vary significantly based on the key assumptions of the analysis and the risk aversion of the investor. Financial advisors will need to discuss in detail their client's risk strategy. In particular, the 20% buffer strategy had relatively significant allocations across a variety of scenarios. Additional context for these allocations is provided in section V.

V. BUFFERS VS. FLOORS

The buffer-based PWSs received significantly higher allocations than the floor-based PWSs considered in the analysis. It is worth providing additional context for this effect. The differences in allocations are largely based on the differences generated from the respective options premium coupled with how the approaches reshape the expected return distribution.

When implementing a buffer strategy an investor is effectively buying risk, since investors are exposing themselves to negative tail risk (beyond whatever the buffer is). Put options, especially out-of-the-money put options

EXHIBIT 12: *Protected Wealth Strategy Allocations with Varied Assumptions, Implied Volatility = 25%*

			0% FLOOR				10% FLOOR				10% BUFFER				20% BUFFER			
%Change			Risk Aversion				Risk Aversion				Risk Aversion				Risk Aversion			
EqΔ	RfΔ	OptΔ	2	4	8	20	2	4	8	20	2	4	8	20	2	4	8	20
4	0	0	3	4	5	2	0	0	0	0	15	16	10	4	44	51	45	26
0	0	0	1	3	5	8	2	1	1	0	37	43	32	14	45	64	64	44
4	0	0	0	2	3	7	0	3	1	0	24	48	48	25	9	43	65	55
-4	1	0	0	0	0	0	0	0	0	0	9	11	6	2	37	37	30	15
0	1	0	0	0	0	0	0	0	0	0	34	36	25	10	42	59	52	33
4	1	0	0	0	0	0	0	0	0	0	24	45	43	21	9	39	59	47
-4	2	0	0	0	0	0	0	0	0	0	5	5	3	1	13	15	11	5
0	2	0	0	0	0	0	0	0	0	0	20	23	15	6	28	35	34	17
4	2	0	0	0	0	0	0	0	0	0	22	36	35	15	5	27	41	32
-4	0	1	27	49	74	90	4	3	1	1	26	23	14	6	66	63	51	28
0	0	1	11	34	64	88	3	5	3	1	60	53	36	15	61	78	71	46
4	0	1	2	16	47	82	0	4	5	2	46	65	53	26	24	60	78	59
-4	1	1	4	4	2	1	1	0	0	0	20	17	9	4	65	54	40	19
0	1	1	2	5	8	7	3	3	2	1	60	48	31	12	61	76	64	37
4	1	1	0	2	9	10	0	3	2	1	46	64	50	23	24	60	75	53
-4	2	1	0	0	0	0	0	0	0	0	13	10	5	2	39	33	22	9
0	2	1	0	0	0	0	0	0	0	0	48	38	23	9	59	64	51	25
4	2	1	0	0	0	0	0	0	0	0	46	59	44	19	24	56	65	40
-4	0	2	54	75	88	95	8	5	3	1	33	26	15	6	78	72	55	29
0	0	2	33	58	83	95	6	8	6	2	68	57	38	16	74	86	75	47
4	0	2	5	31	65	92	2	10	10	4	61	71	55	27	42	71	83	60
-4	1	2	54	75	85	69	5	2	1	0	27	21	11	4	77	66	46	21
0	1	2	33	58	83	88	4	6	3	1	68	53	33	13	74	84	69	38
4	1	2	5	31	66	91	2	7	5	2	61	69	52	23	42	71	82	55
-4	2	2	4	4	2	1	1	0	0	0	18	13	6	3	61	47	30	12
0	2	2	2	6	9	4	4	3	1	1	61	44	26	10	74	78	59	28
4	2	2	1	6	8	6	1	5	3	1	61	65	47	19	42	71	75	43
Average			9	17	26	31	2	3	2	1	36	39	28	12	45	58	55	34

that are going to be used to implement the strategy, are a relatively expensive form of insurance. Since the buffer approach involves selling puts, it can capture this effect. In contrast, although the floor strategy does buy some risk (e.g., selling at-the-money put option), it also requires buying an out-of-the-money put option. This out-of-the-money option is relatively expensive (when considering how implied volatility changes across strike prices), which reduces the overall premium associated with the approach.

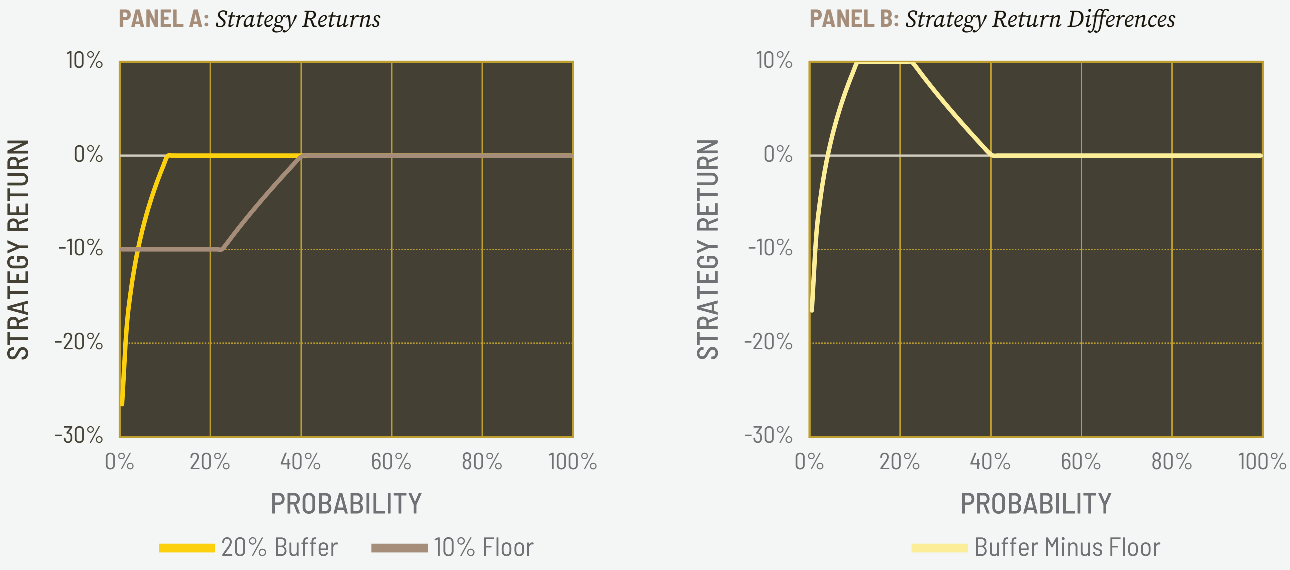
We demonstrate this effect for the 10% floor and 20% buffer strategies in exhibit 13. We select these two PWSs, among the four considered, since they have a relatively similar cap rate using the base assumptions of the analysis. For this section, we assume an average (price) return of 5% and a standard deviation of 20%. We ignore the premium generated from the respective approaches in exhibit 13, since that will vary based on the key assumptions within Black-Scholes (e.g., implied volatility); we address these next.

The underlier (price return) is negative only approximately 40% of the time, which is why the 20% buffer and 10% floor strategies are mostly zero in panel A (i.e., for ~60% of outcomes). As demonstrated in exhibit 9, the 20% buffer strategy is going to have a return that is worse than the 10% floor strategy only when the underlier has a return that is less than –30%.

Returns lower than –30% are relatively rare. Assuming an average return of 5% and a standard deviation of 20%, this would occur in only approximately 4% of scenarios. Focusing on calendar-year returns of the S&P 500 from 1872 to 2020, based on data obtained from Robert Shiller's website (Shiller n.d.), the price return of the S&P 500 has been less than –30% in only 5 out of the 149 years available, which is a 3.4% of the time.

If we assume implied volatility for at-the-money options is 25%, the implied volatility for a put option used for a 20% buffer strategy would be closer to 30%. Assuming

EXHIBIT 13: *Exhibit 13. 20% Buffer Strategy vs. 10% Floor Strategy*



an average return of 5% and a standard deviation of 30% (consistent with the implied volatility in options pricing) the probability of the price return of the S&P 500 being less than –30% is approximately 12%. This is more than three times the historical average and suggests that out-of-the-money put options have historically been relatively expensive (i.e., overpriced the risk associated with a significant negative market return).

While the return of 20% buffer strategy return can obviously be much lower than the 10% floor, that does not necessarily mean the buffer strategy is less efficient, because instead of owning the buffer the investor could have owned the underlier (S&P 500) directly instead. In other words, an investor can have exposure to the tail risk in different ways. The key is to create a strategy that optimally considers the potential benefits of using options combined with other potential risks of the portfolio. By considering options—in other words, a PWS—an investor increases the opportunity set to potentially increase the efficiency of the portfolio.

Even if the realized standard deviation is assumed to increase, or the negative tail is assumed to have additional skewness or kurtosis, the average return of the 20% buffer is going to be higher than the 10% floor strategy, from approximately 30% to 40% of scenarios (where the return of the underlier is negative). This results in an average negative return (the premium is not included so the average return will be negative) for the 20% buffer strategy that is approximately –1.0% versus –3.1% for the 10% floor strategy.

While the 10% floor strategy may result in a higher premium than the 20% buffer strategy (depending on pricing assumptions), the higher premium is not generally enough to offset the lower expected average return. This effect is demonstrated in exhibit 14, which updates the previous analysis (in exhibit 13) by including the expected premium generated by the respective strategies (in addition to the returns) where the options are priced assuming a risk-free rate of 1% and implied volatility level of 20% (panel A) and 25% (panel B). The results in exhibit 14 include the differences in the 20% buffer strategy versus the 10% floor strategy (i.e., how much better is the buffer versus the floor). The analysis effectively assumes a risk-aversion level (γ) of 0.

If implied volatility is only 20%, the return of the 20% buffer strategy typically exceeds the return of the 10% floor strategy (even after considering the premium differences), as long as realized volatility is less than 25%. For higher levels of implied volatility (e.g., panel B, where implied volatility equals 25%) the 20% buffer strategy effectively dominates the 10% floor strategy, which is consistent with the findings of the overall analysis.

The analysis in exhibit 14 focuses on wealth and does not consider risk aversion. While the 10% floor strategy becomes more attractive for higher risk-aversion levels with lower equity returns and higher equity standard deviations, these are also scenarios where the risk-free asset would dominate the PWSs. In other words, the reason the 10% floor strategy does not receive higher allocations when it becomes more attractive than the 20% buffer strategy is because neither PWS receives an allocation in those scenarios.

The 10% floor strategy can become attractive, though, if realized returns and volatility differ significantly from the key assumptions used as part of the optimization. For example, if implied volatility is 20% and the portfolio optimization assumes the price return on equities is going to be 5% with a standard deviation of 20%, an investor with a risk aversion coefficient (γ) of 4 would be expected to have an allocation to a 10% floor strategy that exceeds 50% and a 0% allocation to a 20% buffer strategy (assuming they are mutually exclusive). If, however, the realized return ends up being 2.5% and realized volatility ends up being 25%, the investor would have been (significantly) better off allocating to the 10% floor strategy than to the 20% buffer strategy.

In other words, although the buffer strategy fared better than the floor strategy in this analysis, the returns are assumed to be known when making the allocation decision. While there is some assumed uncertainty given the resampling routine, the approach does not necessarily capture the true uncertainty when it comes to investing (e.g., you might assume stocks are going to go up 5%, but they also might go down 20%).

Therefore, given the fundamental uncertainty associated with investing, a floor strategy has obvious behavioral appeal to some investors and potential additional appeal based on the actual market returns (especially the 0% floor strategy when the options budget is increased).

EXHIBIT 14: *Return Differences in the 20% Buffer Strategy vs. 10% Floor Strategy*

PANEL A: *Implied Volatility = 20%*

REALIZED RETURN	REALIZED VOLATILITY 15%	20%	25%	30%
-5.0%	2.33	0.96	-0.63	-2.34
-2.5%	2.03	0.99	-0.38	-1.93
0.0%	1.66	0.94	-0.20	-1.60
2.5%	1.26	0.83	-0.09	-1.32
5.0%	0.85	0.68	-0.04	-1.10
7.5%	0.45	0.49	-0.03	-0.94
10.0%	0.08	0.29	-0.06	-0.81
12.5%	-0.24	0.08	-0.11	-0.72
15.0%	-0.52	-0.13	-0.19	-0.67

PANEL B: *Implied Volatility = 25%*

REALIZED RETURN	REALIZED VOLATILITY 15%	20%	25%	30%
-5.0%	3.89	2.52	0.94	-0.78
-2.5%	3.59	2.55	1.18	-0.37
0.0%	3.22	2.50	1.36	-0.04
2.5%	2.82	2.39	1.47	0.24
5.0%	2.41	2.24	1.52	0.46
7.5%	2.01	2.05	1.53	0.63
10.0%	1.64	1.85	1.51	0.75
12.5%	1.32	1.64	1.45	0.84
15.0%	1.04	1.43	1.37	0.89

Therefore, although the buffer approach may appear to be better than the floor strategy, the floor strategy still warrants consideration depending on return expectations, risk-aversion levels, and so on.

CONCLUSIONS

This paper explored the potential benefit of approaches that capture market upside while limiting losses, something we call protected wealth strategies (PWSs). The optimal allocation to these products was determined using a resampled optimization framework based on utility theory. While optimal allocations varied significantly across simulations, PWSs were clearly an attractive option for moderately risk-averse investors under certain circumstances. Strategies that involve selling out-of-the-money puts (i.e., buffer approaches), were especially attractive given historical options pricing dynamics.

Individuals are more than capable of implementing the PWSs considered in this analysis. Alternatively, an investor could consider a prepacked protected wealth strategies (P-PWSs), such as fixed indexed annuities (FIAs) or registered index-linked annuities (RILAs). While P-PWSs are likely to have a number of things that should be considered before purchase, such as a lack of liquidity for some term (e.g., 5 years), P-PWSs are often able to offer more-attractive terms (e.g., caps) than an individual household would be able to generate by itself.

While the analysis noted significant potential allocations to PWSs in certain scenarios, one drawback of this analysis was that the underlier for the options strategy was the same as the risk asset available in the portfolio (i.e., equities, as proxied by the S&P 500). While the S&P 500 is perhaps the most widely used underlier for PWSs, future research should explore the implications of considering different underliers, especially those with lower correlations to the investment opportunity set, as well the as the potential role of PWSs as an approach to generate retirement income, since the target investors of these strategies are generally those nearing or in retirement (e.g., older investors who are moderately risk averse).

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APPENDIX 1. BLACK-SCHOLES PRICING MODEL

The Black-Scholes pricing model is a partial differential equation that can be used to describe the price of a European option over time. While it was originally introduced for investments without payouts, it can easily be extended to allow for the fact that the underlying may have payouts during the life of the option (i.e., dividends). Equation A1 is noted below:

EQUATION A1

$$C(S_0,t)= e^{-r(T-t)}\,(FN(d_1) - KN(d_2))$$

and

$$P(S_0,t)= e^{-r(T-t)}(KN(-d_2) - FN(-d_1))$$

where now

$$F = S_0 e^{(r-q)(T-t)}$$

is the modified forward price that occurs in the terms d_1 and d_2:

$$d_1=\frac{\ln\left(\frac{F}{K}\right)+(\sigma^2/2)(T-t)}{\sigma\sqrt{T-t}}$$

and

$$d_2= d_1- \sigma\sqrt{T-t}$$

where

C = price of a European call option

P = price of a European put option

K = strike price of option

t = time in years until expiration (current)

T = original time until expiration in years

F = modified forward price

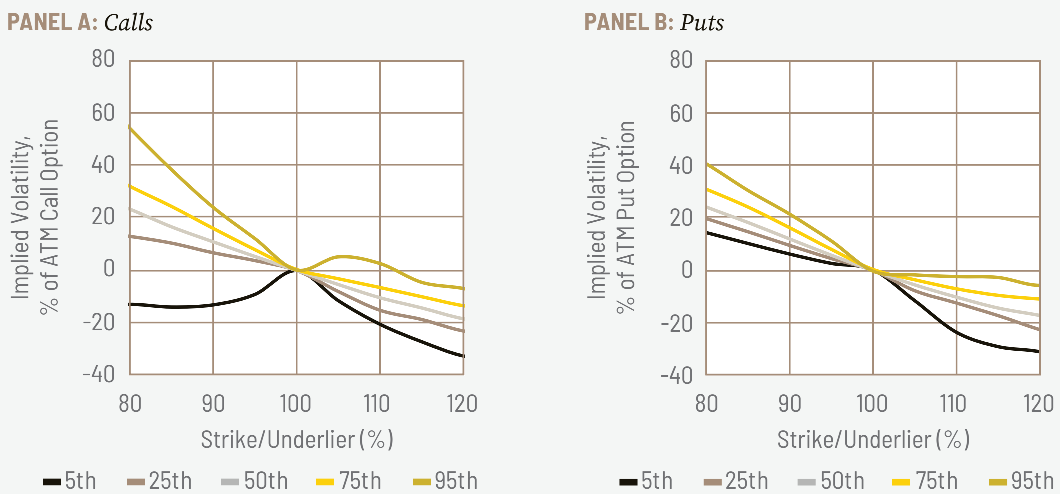
r = continuously compounded risk-free rate, expressed as an annual percentage

q = continuously compounded dividend yield

σ = annual standard deviation (volatility) of underlying asset

Within the actual options market each of the inputs are directly observable expect for implied volatility, which can be implied using an options pricing formula (e.g., Black-Scholes) and the other known inputs. It is worth noting that implied volatility is not constant across strike prices or the time until expiration. Implied volatilities form what is known as an implied volatility surface. We demonstrate how implied volatility has varied as a percentage of the at-the-the money implied volatility estimate using historical data on the S&P 500 Index obtained from DeltaNeutral.com from 1990 to 2020 in exhibit A1.1.

EXHIBIT A1.1: *Differences in Implied Volatility Estimates vs. at-the-Money Strike Options, 1990–2020*



In-the-money calls and out-of-the-money puts have higher levels of implied volatility, but the median differences historically are very similar between puts and calls. While there clearly have been variations in how implied volatility changes across strike prices, we use equation A1.1 to adjust the at-the-money implied volatility (IV_{ATM}) to the assumed level of implied volatility (IV_s) for a given strike price (Sp) and underlier price (U_p) for the analysis.

EQUATION A1.1

$$IV_s = IV_{ATM} * \left(\left(\left(\frac{Sp}{Up} \right)^2 * .00796 \right) - \left(\left(\frac{Sp}{Up} \right) * 2.628 \right) + 183.151 \right)$$

The risk-free rate is for the Black-Scholes equation is typically tied to an instrument like the LIBOR; however, we use 1-year government bond yields as a simplifying assumption, effectively assuming the two are equal. Historical yields on 1-year Treasuries and 1-year LIBOR have been relatively similar, with a correlation of .993 from January 1986 to December 2020 (see exhibit 4).

APPENDIX 2. OPTIONAL ALLOCATIONS BY RISK AVERSION LEVEL FOR PROTECTED WEALTH STRATEGIES

EXHIBIT A2.1: *Optimal Allocations by Risk Aversion Level for Protected Wealth Strategies, ATM Implied Volatility = 20%*

			0% FLOOR				10% FLOOR				10% BUFFER				20% BUFFER			
% Change			Risk Aversion				Risk Aversion				Risk Aversion				Risk Aversion			
EqΔ	RfΔ	OptΔ	2	4	8	20	2	4	8	20	2	4	8	20	2	4	8	20
-4	0	0	3	4	5	5	3	1	1	0	2	6	5	2	13	21	24	16
0	0	0	1	3	5	8	2	3	2	1	9	17	19	9	6	24	37	32
4	0	0	0	2	5	11	0	3	2	1	0	18	33	20	0	11	37	42
-4	1	0	0	0	0	0	0	0	0	0	2	3	2	1	3	7	9	5
0	1	0	0	0	0	0	0	0	0	0	7	11	12	5	3	10	21	17
4	1	0	0	0	0	0	0	0	0	0	0	11	24	14	0	4	19	28
-4	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	2	0	0	0	0	0	0	0	0	0	4	7	5	2	0	0	0	0
4	2	0	0	0	0	0	0	0	0	0	0	7	15	7	0	0	1	0
-4	0	1	28	51	75	91	7	5	2	1	15	16	10	4	35	44	41	24
0	0	1	13	35	66	89	4	7	5	2	37	43	32	14	36	54	59	42
4	0	1	2	17	49	83	1	7	9	4	24	48	48	25	6	34	60	53
-4	1	1	6	4	5	2	4	2	1	0	9	11	6	2	26	28	25	12
0	1	1	4	9	8	9	3	5	3	1	34	36	25	10	33	49	45	30
4	1	1	0	2	9	12	1	4	4	2	24	45	43	21	6	30	52	43
-4	2	1	0	0	0	0	0	0	0	0	5	5	3	1	7	11	8	3
0	2	1	0	0	0	0	3	2	1	0	20	23	15	6	13	21	26	13
4	2	1	0	0	0	0	1	4	2	1	22	36	35	15	2	21	32	27
-4	0	2	58	77	89	95	17	12	7	3	33	26	15	6	66	63	51	28
0	0	2	35	61	84	95	18	20	15	6	68	57	38	16	61	78	71	46
4	0	2	7	33	66	93	13	30	25	10	61	71	55	27	24	60	78	59
-4	1	2	58	77	88	73	12	7	4	1	27	21	11	4	65	54	40	19
0	1	2	35	61	84	90	18	15	9	3	68	53	33	13	61	76	64	37
4	1	2	7	33	68	93	13	22	15	6	61	69	52	23	24	60	75	53
-4	2	2	10	12	8	3	5	3	1	0	18	13	6	3	39	33	22	9
0	2	2	8	17	17	9	11	8	4	2	61	44	26	10	59	64	51	25
4	2	2	2	10	17	13	11	13	8	3	61	65	47	19	24	56	65	40
Average			10	19	28	32	5	6	4	2	25	28	23	10	23	34	37	26